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A New Weight-Restricted DEA Model Based on PROMETHEE II

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Abstract

Weights restrictions in DEA[1] were initiated with the goal of making DEA outputs more reasonable. In classic DEA models, some inputs/outputs may be characterized by low or high weight values. These can be in contradiction with a priori information leading to counterintuitive interpretations. The aim of this paper is to investigate a new weight restricted DEA approach based on a MCDA[2] methodology. To achieve this goal we use the stability intervals based on the PROMETHEE II as weight constraints in DEA. As expected, these restrictions improve the discrimination power of the model. In the new approach, the unicriterion net flow scores matrix is used instead of the initial evaluation matrix. As well, we already integrate preferential information in the DEA process. By construction, the best results are compatible with the PROMETHEE[3] II ranking. Additional comparisons with the outputs of other decision making techniques are provided in three real examples.

Keywords: Data Envelopment Analysis, Multiple Criteria Decision Aid, PROMETHEE, Stability Intervals, weight restrictions

1. DEA= Data Envelopment Analysis
2. MCDA= Multiple Criteria Decision Aid
3. PROMETHEE= Preference Ranking Organization METHod for Enrichment of Evaluations
4. DM= Decision Maker
5. AHP= Analytic Hierarchy Process
6. AR= Assurance Region
7. DMU= Decision Making Unit
8. CCR= Charnes, Cooper and Rhodes
9. BCC= Banker, Charnes and Cooper
10. CRS= Constant Return to Scale
11. VRS= Variable Return to Scale
12. MAUT= Multiple Attribute Utility Theory
13. ELECTRE= ELimination Et Choix Traduisant la REalité (ELimination and Choice Expressing REality)
14. WSI= Weight Stability Interval
15. WCCR= Weighted CCR model
16. PIIWCCR= PROMETHEE II Weighted CCR
17. PIIWBCC= PROMETHEE II Weighted BCC
18. RCCR= Reduced CCR
19. SE= Super Efficient
20. IWEPS= Institut Wallon de l'Evaluation, de la Prospective et de la Statistique

1. Introduction

This paper presents a new DEA-MCDA integrated model. The aim is to restrict weight values in Data Envelopment Analysis (DEA) by using tools from MCDA (Multiple Criteria Decision Aid).

The automatic generation of weights is one of the main distinctive features of the original DEA methods. Nevertheless some researchers have pointed out critics about the use of DEA without a priori information regarding the weights [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11] and [12]. In the original DEA method, weights variations are unrestricted. In some cases, this flexibility causes understanding problems: some inputs or outputs can be characterized by arbitrarily low or high values leading to counterintuitive interpretations. Furthermore, the computed values may be in contradiction with a priori information offered by the Decision Maker (DM[4]) (since he/she believes that weights belong to some pre-determined intervals). During the last 3 decades, researchers have proposed different techniques to address this problem. We refer the interested reader to Thompson et al. [1], [2] and [3], Dyson and Thanassoulis [4], Wong and Beasley [5], Roll and Golany [6] and [7], Allen [8], Takamura and Tone [9], Ueda [10] and [11] and Dimitrov and Sutton [12].

Thompson et al. [1] were among the first researchers to suggest the use of weight restrictions in DEA. They tackled the problem of assessing the efficiency of a group of physics laboratory. With classical models, all units were efficient. When applying weight restrictions, they showed that a single unit appeared to be efficient. Dyson and Thanassoulis [4] attempted to eliminate the use of zero weights by applying regression analysis. Wong and Beasley [5] proposed the so-called virtual weights model when imposing weight restrictions in DEA based on lower and/or upper bounds. Unfortunately, the large number of restrictions that should be imposed on the weights can increase the possibility of infeasibility (in the optimization problem) [13]. Roll and Golany [7] run a basic DEA model to use its generated weights in determining bounds. Takamura and Tone [9] used the judgments of people who are familiar with the characteristics of the evaluated objects to propose weights restrictions. Ueda [10], [11] suggested a canonical correlation analysis to set upper and lower bounds. Dimitrov and Sutton [12] proposed a symmetric weight assignment technique.

To our point of view, restricting weights values in DEA models is a first step to integrate the preferences of the DM in the evaluation process. The synergies between MCDA and DEA follow this trend. Shang et Sueyoshi [14], Sinuany-Stern et al. [15], Liu [16] and Takamura and Tone [9] are researchers who investigated the combination of AHP[5] (Analytic Hierarchy Process) [17] and DEA. The works of Liu [16] are focused on the integration of two subjective and objective weight restrictions method. Two weights were derived for each input/output: one from AHP and other from a classical DEA model. The author created a new weight by mixing these two values and ranked the Decision Making Units (DMUs[7]). Takamura and Tone [9] integrated Assurance Region (AR[6]) [2] and AHP. Junior [18] used MACBETH (Bana et Costa et al., 2003) [19] as a MCDA tool to give restrictions to DEA weights in the Wong and Beasley virtual weight restrictions DEA model [5]. MACBETH is initially used to build a judgment matrix by asking the DMs to evaluate the variety of attractiveness among each pair of actions by choosing one of these categories: indifference, very weak, weak, moderate, strong, very strong and extreme. On the basis of this matrix, desired weights are generated. The minimum and maximum of these values are set as lower and upper bounds to use in the virtual weight restrictions DEA model. Let us note that, in some cases, this method can be in contradiction with the original MACBETH ranking. To avoid this weakness, Junior, added some additional constraints to the virtual weight restrictions. Unfortunately, as stated by Lins et al. [13], the large number of constraints can increase the likelihood of infeasibility.

Determining subjective bounds for weight values is usually a complex cognitive task for DMs. In this work we apply PROMETHEE (Brans J.P. 1982) [20] as a MCDA tool to automatically determine bounds for DEA weights. Then, we solve a DEA model without inputs, where the outputs are unicriterion net flow scores and the objective function is to maximize the PROMETHEE II net flow scores. The computed lower and upper weight bounds are derived from stability intervals of PROMETHEE II. In our perspective, the fewer number of weight constraints in our model (2*No. Criteria) compared to the virtual weight restrictions DEA model (2*No. Criteria*No. DMUs) [5], decreases the likelihood of infeasibility (Lins et al., 2007) [13].

The proposed Weight Restrictions DEA model is based on the unicriterion net flow scores. Therefore, it already integrates preferential information given by the DM. By construction, the method is closely related to the PROMETHEE II ranking. In what follows, we will indeed show that:

1) The best alternatives remain the same in the two approaches;
2) There exists also a correlation between the two methods.

Additionally, we also investigate the correlation of the proposed method outputs with other classic DEA models. Finally, we show that, as expected, the discrimination power of the model is enhanced. Further, it is worth noting that, to the best of our knowledge, this contribution is the first to investigate the combination of PROMETHEE and DEA. PROMETHEE methods are known for their simplicity and their ease of use. Applying them to impose weight restrictions to DEA and to further discriminate the alternatives in the DEA analysis, should facilitate the DM's interpretation.

The paper is organized as follows: in the second section, a brief outline of DEA basic models is provided. It is followed by a brief description of PROMETHEE II and the stability intervals method in section three. The fourth section is dedicated to the presentation of the Weight Restrictions DEA method. Finally, in the section five, we illustrate the model in three numerical examples.

2. Data Envelopment Analysis

In this section, we briefly outline the basic concepts of DEA. Of course, a complete description goes beyond the scope of this paper. We refer the interested reader to [21], [22] for a complete analysis.

DEA was first introduced in 1978 by Charnes, Cooper and Rhodes (CCR[8]) [23]. They explained how a fractional linear measure of outputs compared to inputs (or reversely inputs compared to outputs) could be computed in order to evaluate the relative efficiency of so-called Decision Making Units (DMUs). DEA is a non-parametric and non-statistical method (Doyle et Green, 1993) [24] that is based on the resolution of a Linear Program (LP) for each DMU. Weights of different factors are determined automatically in order to find the best possible efficiency score. The method works in two steps. First, it determines an efficient frontier by identifying DMUs belonging to it. Then, the relative efficiency of other DMUs (inefficient units) is estimated based on their deviation from this frontier.

Let E_o denotes the efficiency measure of DMU_o. This DMU is supposed to be characterized by s outputs, $y_{ro}, r = 1,2, \dots, s$ and m inputs, $x_{io}, i = 1,2, \dots, m$. We have:

$$\boldsymbol{E_o = Max\ \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}} \tag{1}$$

While u_r and v_i are non-negative weights.

If we further assume that:

1) The efficiency of DMUs has to be less than or equal to 1;
2) The sum of weighted inputs in objective function equal to 1;

We obtain the traditional CCR model [23]:

$$\boldsymbol{E_o = Max \sum_{r=1}^{s} u_r y_{ro}}$$

Such that

$$\boldsymbol{\sum_{i=1}^{m} v_i\, x_{io}=1,} \tag{2}$$

$$\boldsymbol{\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} \leq 0, j = 1, 2, \dots, o, \dots, n}$$

$$\boldsymbol{u_r\, , v_i \geq 0, \forall\ i, r}$$

This problem is solved n times to calculate the efficiency score of DMU_j , $j = 1,2, \dots, o, \dots, n$. In this context, it is worth noting that the production function is assumed to satisfy the Constant Return to Scale (CRS[10]) assumption. In order to modify it to Variable Return to Scale (VRS[11]) [25] a constant variable (u_O) should be added as follows (BCC[9] formulation [26]):

$$\boldsymbol{E_o = Max \sum_{r=1}^{s} u_r y_{ro} + u_O}$$

Such that

$$\boldsymbol{\sum_{i=1}^{m} v_i\, x_{io}=1,} \tag{3}$$

$$\boldsymbol{\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} + u_O \leq 0, j = 1, 2, \dots, o, \dots, n}$$

$$\boldsymbol{u_r\, , v_i \geq 0, \forall\ i, r, u_O\ free\ in\ sign}$$

These formulations are input-oriented multiplier models [21]. In other words, the LP is formulated so as to determine how to improve the input features of a unit to become efficient in order to achieve the same output level.

Finally, let us point out the Super Efficient (SE[19]) model to generate a complete ranking of DMUs. It was developed by Anderson and Peterson in 1993 [27] in order to rank the efficient DMUs (that were equivalent in previous models). The technique removes the oth constraint in the formulation to enable an extreme efficient unit o to achieve an efficiency score that could be greater than 1. It can be appeared as follows:

$$\boldsymbol{E_o = Max \sum_{r=1}^{s} u_r y_{ro}}$$

Such that

$$\boldsymbol{\sum_{i=1}^{m} v_i \, x_{io} = 1,} \tag{4}$$

$$\boldsymbol{\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i \, x_{ij} \ \leq 0, j = 1, 2, \dots, o, \dots, n, j \neq o}$$

$$\boldsymbol{u_r \, , v_i \geq 0, \forall \, i, r}$$

These models are used in section 5 in order to compare the results with the proposed method.

3. PROMETHEE II

During the last five decades, several Multiple Criteria Decision Aid (MCDA) techniques like MAUT[12] (Keeney and Raiffa, 1976) [28], AHP (Saati, 1980) [17], ELECTRE[13] (Roy, 1991) [29] and PROMETHEE[3] (Brans, 1982) [20] have been developed with the aim of supporting DMs in the selection of compromise solution(s) and the ranking or sorting of alternatives. In this paper, we focus ourselves on PROMETHEE II (Preference Ranking Organization Method for Enrichment Evaluations). The family of PROMETHEE methods is known thanks to their simplicity, an important number of applications in different fields such as finance, business, education, health care, etc. (Behzadian, M., 2010) [30] and the existence of user friendly software, D-sight (Hayez, Q. et al., 2012) [31].

The PROMETHEE method has been developed by J. P. Brans in 1982 [20; 32] and is based on pair wise comparisons. PROMETHEE II allows a DM to rank a finite set of n actions (DMUs in DEA) $A = \{a_1, \dots a_j, \dots, a_n\}$ that are evaluated over a set of q criteria $F = \{f_1(a_j), \dots, f_k(a_j), \dots, f_q(a_j)\}$. Let $f_k(a_j)$ denotes the evaluation of action a_j on criterion $f_k, k = 1, \dots, q$. In what follows, we assume (without loss of generality) that criteria have to be maximized.

The first step of the method consists to compute differences between every pair of actions on all criteria. More formally:

$$\boldsymbol{d_k(a_i, a_j) = f_k(a_i) - f_k(a_j), \forall a_i, a_j \in A, \forall k = 1, \dots, q} \tag{5}$$

This MCDA method works in the framework of preference functions to integrate intra-criterion information. Therefore one has to associate a generalized criterion $\{f_k(.), P_k(a_i, a_j)\}$ to each criterion. $P_k(a_i, a_j)$ provides the preference strength of action a_i over a_j. It is assumed to be a positive non-decreasing function of $d_k(a_i, a_j)$. The concept of preference function is used to transform the difference into a unicriterion preference degree; hence:

$$\boldsymbol{\pi_k(a_i, a_j) = P_k[d_k(a_i, a_j)]} \tag{6}$$

The method provides the DM with a set of predefined preference functions for which at most two parameters have to be defined (the indifference and preference thresholds). Details about preference functions are explained in (Brans and Mareschal, 2002) [33].

Usually, 6 main different types of preference functions are used. In this paper we limit ourselves to linear functions:

$$\pi_k(a_i, a_j) = \begin{cases} 0 & d_k(a_i, a_j) \le q_k \\ \dfrac{d_k(a_i, a_j) - q_k}{p_k - q_k} & q_k < d_k(a_i, a_j) \le p_k \\ 1 & d_k(a_i, a_j) > p_k \end{cases}$$

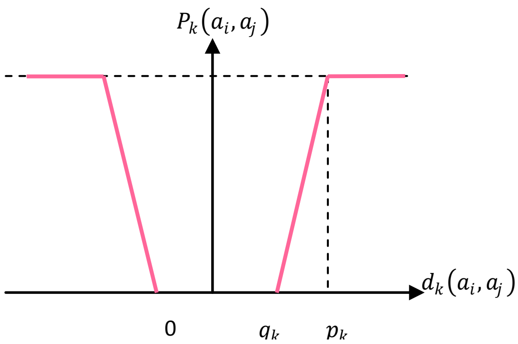

Figure 1- Linear preference function. q_k and p_k are respectively the indifference and preference thresholds associated to criterion k. If $d_k \in [0, q_k]$, a_i, a_j are considered indifferently on criterion k and if d_k is greater than p_k, a_i is strictly prefered to a_j. Between these 2 thresholds, the preference function is assumed to linearly increase.

The global preference degree between a_i and a_j is computed as follows:

$$\boldsymbol{\pi(a_i, a_j) = \sum_{k=1}^{q} \pi_k(a_i, a_j). w_k} \tag{7}$$

Where $w_k, k = 1, \dots, q$ are normalized positive weights associated to the different criteria. This degree varies between 0 and 1. Obviously we have:

$$\boldsymbol{\pi(a_i, a_j) \ge 0 \quad , \quad \pi(a_i, a_j) + \pi(a_j, a_i) \le 1} \tag{8}$$

The positive and negative outranking flows are defined as follows:

$$\emptyset^+(\boldsymbol{a_j}) = \frac{\boldsymbol{1}}{\boldsymbol{n-1}} \sum_{\boldsymbol{x \epsilon A}} \boldsymbol{\pi(a_j, x)}$$

$$\emptyset^-(\boldsymbol{a_j}) = \frac{\boldsymbol{1}}{\boldsymbol{n-1}} \sum_{\boldsymbol{x \epsilon A}} \boldsymbol{\pi(x, a_j)} \tag{9}$$

The PROMETHEE I ranking is the intersection of the two rankings induced by these flows. Moreover, a complete pre-order, called PROMETHEE II can be obtained on the basis of the net flow score:

$$\emptyset(\boldsymbol{a_j}) = \emptyset^+(\boldsymbol{a_j}) - \emptyset^-(\boldsymbol{a_j}) \tag{10}$$

Let us stress that the net flow score can also be computed as follows:

$$\emptyset(\boldsymbol{a_j}) = \sum_{\boldsymbol{k=1}}^{\boldsymbol{q}} \emptyset_{\boldsymbol{k}}(\boldsymbol{a_j}). \boldsymbol{w_k} \tag{11}$$

Such that:

$$\emptyset_k(a_j) = \frac{1}{n-1}\sum_{x\epsilon A}\{\pi_k(a_j, x) - \pi_k(x, a_j)\}, \quad k = 1, \dots, q \tag{12}$$

The quantity $\emptyset_k$ is called the unicriterion net flow score of action a_j and is such that $-1 \le \emptyset_k(a_j) \le 1$. At this point, it is worth noting that the multicriteria problem can be viewed as an evaluation table (and associated parameters) or a matrix $\emptyset = (\emptyset_k(a_j))$. These values already integrate intra-criterion parameters and are all lying in the same range.

The determination of exact weight values is often a cognitive complex task for the DM. When the ranking is computed a natural question can be raised: what is the impact of changing a given weight value? The purpose of the Weight Stability Intervals (WSI[14]) technique (Mareschal, B., 1988) [34] is to preserve the preference ranking of a subset of alternatives within the intervals of criteria weights. Its main strength is the automated generation of intervals limits that confirms the robustness of PROMETHEE II outputs (typically maintaining the rank of the first alternative). A complete description of this method goes beyond the scope of this paper. In what follows, we will only sketch the main steps. Moreover, we will only consider the case of preference between two alternatives $a_i \, P \, a_j \leftrightarrow \emptyset(a_i) > \emptyset(a_j)$.We refer the interested reader to [34] for complete explanations.

In a complete preorder the structure of (P, I) between a pair of actions (a_i, a_j) is:

$$\begin{cases} a_i \, P \, a_j \quad iff \quad \emptyset(a_i) > \emptyset(a_j) \\ a_i \, I \, a_j \quad iff \quad \emptyset(a_i) = \emptyset(a_j) \end{cases} \tag{13}$$

In this context, we want to investigate the potential modification in the (P, I) structure when the weights are changed. Let us state $w_k^{'} = (1+\beta).w_k$ (with $\beta \ge -1$) for criterion f_k. To maintain the normalization of modified weights, all the other weights should be rearranged as $w_l^{'} = \alpha.w_l$ when $l \ne k$. The parameters α and β are related as follows:

$$\alpha = \frac{1-(1+\beta)w_k}{1-w_k} \tag{14}$$

Constraints on α and β guarantee the non negativity of the modified weights:

$$-1 \le \beta \le \frac{1-w_k}{w_k} \tag{15}$$

$$0 \le \alpha \le \frac{1}{1-w_k} \tag{16}$$

As a consequence, w_k can be increased or decreased from its initial value according to the values of α and β. Let us denote $(P^{'}, I^{'})$ the complete preorder of modified weights $(w_k^{'})$. We have:

$$\Delta(a_i, a_j) = \emptyset(a_i) - \emptyset(a_j) \tag{17}$$

$$\Delta^{'}(a_i, a_j) = \emptyset^{'}(a_i) - \emptyset^{'}(a_j) \tag{18}$$

$$\Delta_k(a_i, a_j) = \emptyset_k(a_i) - \emptyset_k(a_j) \tag{19}$$

It is easy to show that:

$$\Delta^{'}(a_i, a_j) = \alpha\Delta(a_i, a_j) + (1-\alpha)\Delta_k(a_i, a_j) \tag{20}$$

In other words, if we want to hold the relative position between two pairs of actions, we have to satisfy the following constraint:

$$\Delta(\boldsymbol{a_i}, \boldsymbol{a_j})\Delta'(\boldsymbol{a_i}, \boldsymbol{a_j}) > 0 \quad s.t. \quad \Delta(\boldsymbol{a_i}, \boldsymbol{a_j}) \neq \mathbf{0} \tag{21}$$

Thus:

$$\Delta(\boldsymbol{a_i}, \boldsymbol{a_j})[\boldsymbol{\alpha}\Delta(\boldsymbol{a_i}, \boldsymbol{a_j}) + (\mathbf{1} - \boldsymbol{\alpha})\Delta_{\boldsymbol{k}}(\boldsymbol{a_i}, \boldsymbol{a_j})] > 0 \tag{22}$$

$$\boldsymbol{\alpha}[\Delta(\boldsymbol{a_i}, \boldsymbol{a_j})\,\Delta_{\boldsymbol{k}}(\boldsymbol{a_i}, \boldsymbol{a_j}) - \Delta^{\mathbf{2}}(\boldsymbol{a_i}, \boldsymbol{a_j})] < \Delta(\boldsymbol{a_i}, \boldsymbol{a_j})\,\Delta_{\boldsymbol{k}}(\boldsymbol{a_i}, \boldsymbol{a_j}) \tag{23}$$

Let us investigate different cases:

1. When f_k is in disagreement with ranking of a_i and $a_j \leftrightarrow \Delta(\boldsymbol{a_i}, \boldsymbol{a_j})\,\Delta_{\boldsymbol{k}}(\boldsymbol{a_i}, \boldsymbol{a_j}) < 0$, (23) gives the lower bound of α (α_k^-) :

$$\frac{\Delta(\boldsymbol{a_i}, \boldsymbol{a_j})\,\Delta_{\boldsymbol{k}}(\boldsymbol{a_i}, \boldsymbol{a_j})}{\Delta(\boldsymbol{a_i}, \boldsymbol{a_j})\,\Delta_{\boldsymbol{k}}(\boldsymbol{a_i}, \boldsymbol{a_j}) - \Delta^{2}(\boldsymbol{a_i}, \boldsymbol{a_j})} < \boldsymbol{\alpha} \tag{24}$$

This bound has to be satisfied for all pairs of actions that fall in this category. Therefore let us define Ω^- as follows:

$$\boldsymbol{\Omega}^- = \{(\boldsymbol{a_i}, \boldsymbol{a_j}) \in \boldsymbol{A} \times \boldsymbol{A}, \boldsymbol{s.t.}\,\Delta(\boldsymbol{a_i}, \boldsymbol{a_j})\,\Delta_{\boldsymbol{k}}(\boldsymbol{a_i}, \boldsymbol{a_j}) < 0\} \tag{25}$$

We have:

$$\boldsymbol{\alpha_k^-} = \underset{(\boldsymbol{a_i}, \boldsymbol{a_j}) \in \boldsymbol{\Omega}^-}{\boldsymbol{max}} \frac{\Delta(\boldsymbol{a_i}, \boldsymbol{a_j})\,\Delta_{\boldsymbol{k}}(\boldsymbol{a_i}, \boldsymbol{a_j})}{\Delta(\boldsymbol{a_i}, \boldsymbol{a_j})\,\Delta_{\boldsymbol{k}}(\boldsymbol{a_i}, \boldsymbol{a_j}) - \Delta^{2}(\boldsymbol{a_i}, \boldsymbol{a_j})} \tag{26}$$

2. When f_k is in agreement with ranking of a_i and $a_j \leftrightarrow \Delta(\boldsymbol{a_i}, \boldsymbol{a_j})\,\Delta_{\boldsymbol{k}}(\boldsymbol{a_i}, \boldsymbol{a_j}) > \Delta^{\mathbf{2}}(\boldsymbol{a_i}, \boldsymbol{a_j})$, (23) gives the upper bound of α (α_k^+). In this situation it corresponds to:

$$\frac{\Delta(\boldsymbol{a_i}, \boldsymbol{a_j})\,\Delta_{\boldsymbol{k}}(\boldsymbol{a_i}, \boldsymbol{a_j})}{\Delta(\boldsymbol{a_i}, \boldsymbol{a_j})\,\Delta_{\boldsymbol{k}}(\boldsymbol{a_i}, \boldsymbol{a_j}) - \Delta^{2}(\boldsymbol{a_i}, \boldsymbol{a_j})} > \boldsymbol{\alpha} \tag{27}$$

As before, this bound has to be satisfied for all pairs of actions that fall in this category. Therefore let us define $\boldsymbol{\Omega}^+$ as follows:

$$\boldsymbol{\Omega}^+ = \{(\boldsymbol{a_i}, \boldsymbol{a_j}) \in \boldsymbol{A} \times \boldsymbol{A}, \boldsymbol{s.t.}\,\Delta(\boldsymbol{a_i}, \boldsymbol{a_j})\,\Delta_{\boldsymbol{k}}(\boldsymbol{a_i}, \boldsymbol{a_j}) > \Delta^{\mathbf{2}}(\boldsymbol{a_i}, \boldsymbol{a_j})\} \tag{28}$$

Therefore the upper bound of α is:

$$\boldsymbol{\alpha_k^+} = \underset{(\boldsymbol{a_i}, \boldsymbol{a_j}) \in \boldsymbol{\Omega}^+}{\boldsymbol{min}} \frac{\Delta(\boldsymbol{a_i}, \boldsymbol{a_j})\,\Delta_{\boldsymbol{k}}(\boldsymbol{a_i}, \boldsymbol{a_j})}{\Delta(\boldsymbol{a_i}, \boldsymbol{a_j})\,\Delta_{\boldsymbol{k}}(\boldsymbol{a_i}, \boldsymbol{a_j}) - \Delta^{2}(\boldsymbol{a_i}, \boldsymbol{a_j})} \tag{29}$$

3. Finally when $0 \leq \Delta(\boldsymbol{a_i}, \boldsymbol{a_j})\,\Delta_{\boldsymbol{k}}(\boldsymbol{a_i}, \boldsymbol{a_j}) \leq \Delta^{\mathbf{2}}(\boldsymbol{a_i}, \boldsymbol{a_j})$, then inversion between a_i and a_j is not possible. As before let us define:

$$\boldsymbol{\Omega}^{\mathbf{0}} = \{(\boldsymbol{a_i}, \boldsymbol{a_j}) \in \boldsymbol{A} \times \boldsymbol{A}, \boldsymbol{s.t.}\ \Delta(\boldsymbol{a_i}, \boldsymbol{a_j}) = \mathbf{0}\ \boldsymbol{and}\ \Delta_{\boldsymbol{k}} \neq \mathbf{0}\} \tag{30}$$

According to (23) this situation shows that there exist 2 states for set Ω^0:

(I) If $\mathbf{\Omega^0} \neq \emptyset$, then value of α is fixed in 1. It means no change of weights is allowed (no inversions between preferences and only changing from indifference to preference can happen)

(II) If $\mathbf{\Omega^0} = \emptyset$ then $\boldsymbol{\alpha_k^-} < \boldsymbol{\alpha} < \boldsymbol{\alpha_k^+}$.

The final step is determining the WSI of criterion f_k. From (14), β is: $\frac{1-w_k}{w_k}(1-\alpha)$ whereas its stability area is: $\beta_k^- \leq \beta \leq \beta_k^+$. When $W_k^- = (1+\beta_k^-)w_k$ and $W_k^+ = (1+\beta_k^+)w_k$, the WSI is:

$$\boldsymbol{WSI_k = (W_k^-, W_k^+) = (1 - (1 - w_k)\alpha_k^+, 1 - (1 - w_k)\alpha_k^-).} \quad (31)$$

In the next section we use the PROMETHEE II WSI as weight constraints in DEA. Let us note that these conditions can relaxed when the DM wants to focus himself on a subset of alternatives (and not the whole ranking). For instance, he/she can focus himself/herself on the top h alternatives.

4. Methodology

As already stressed, the purpose of using the PROMETHEE II Weight Stability Intervals (WSI) tool in DEA is to restrain full freedom of weights. This allows to avoid undesirable results such as allocating very low or high weight values to some outputs/inputs. In addition preferences of the DM can be partly integrated in the DEA analysis. In this work, we automatically generate the WSI in PROMETHEE II and use these intervals in a CCR (or BCC) input oriented DEA problem. This ensures that the compromise solution(s) identified in PROMETHEE is characterized by an efficient score equal to 1 in DEA.

A first classic weighted DEA model, in the presence of WSI, can be defined as follows:

$$\boldsymbol{E_o = Max \sum_{r=1}^{s} u_r y_{ro}}$$

Such that

$$\boldsymbol{\sum_{i=1}^{m} v_i x_{io} = 1,}$$

$$\boldsymbol{\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, j = 1, 2, \dots, o, \dots, n} \quad (32)$$

$$\boldsymbol{w_i^- \leq v_i \leq w_i^+;\ v_i \geq 0, \forall\, i}$$

$$\boldsymbol{w_r^- \leq u_r \leq w_r^+;\ u_r \geq 0, \forall\, r}$$

Where $w_i^-, w_r^-, w_i^+, w_r^+$ are the PROMETHEE Weight Stability Intervals for the inputs and the outputs. This simple weighted DEA model is referred as WCCR[15] in numerical examples. At this point, it is worth noting that the meaning of weights in this formulation and in PROMETHEE is not the same (nevertheless we will keep WCCR in the next section for illustrative purposes). Therefore, we consider an alternative model where we maximize the net flow scores in a DEA formulation without inputs (when outputs are unicriterion net flow scores of the PROMETHEE II).

Thus (32) is changed to:

$$\boldsymbol{E_o} = \mathbf{Max}\ [\emptyset(\boldsymbol{a_o}) = \textstyle\sum_{k=1}^{q} \boldsymbol{w_k}\emptyset_{\boldsymbol{k}}(\boldsymbol{a_o})]$$

Such that

$$\textstyle\sum_{k=1}^{q} \boldsymbol{w_k}\emptyset_{\boldsymbol{k}}(\boldsymbol{a_j}) \le \mathbf{1};\ \boldsymbol{j} = \mathbf{1}, \mathbf{2}, \dots, \boldsymbol{n} \tag{33}$$

$$\boldsymbol{W_k^-} \le \boldsymbol{w_k} \le \boldsymbol{W_k^+}$$

$$\boldsymbol{w_k} \ge \mathbf{0},\ \forall\ \boldsymbol{k}$$

The constraint (33) imposes that the efficiency scores are less than 1. The presented model is CCR/CRS but it can be easily extended to a BCC/VRS model too. In what follows, we will denote them: PIIWCCR[16] and PIIWBCC[17].

Conceptually this model is an input-oriented multiplier DEA model. Lovelle and Pastor [35] proved that an input-oriented CCR multiplier model without outputs is infeasible. Hence, there is no difficulty to solve our CCR model. Nevertheless, concerning BCC models it can be a bit different. They proved that a BCC input-oriented model without inputs makes no-sense. To solve this problem either it can be transformed to its equivalent output-oriented BCC model or a unit vector can be added to the matrix $\emptyset$ as a single input [21]. As mentioned, the objective function of this problem is the maximization of the PROMETHEE II net flow scores; thus this is done by adding a dummy element $\emptyset_l(a_j) = 1$ (considered as the input l for a_j – this will be the same for all DMUs) in the unicriterion net flow scores matrix.

$$\boldsymbol{E_o} = \mathbf{Max}\ \left(\frac{[\sum_{k=1}^{q} \boldsymbol{w_k}\emptyset_{\boldsymbol{k}}(\boldsymbol{a_o})]}{\boldsymbol{v_l}\emptyset_{\boldsymbol{l}}(\boldsymbol{a_o})}\right) \tag{34}$$

Such that

$$\frac{[\sum_{k=1}^{q} \boldsymbol{w_k}\emptyset_{\boldsymbol{k}}(\boldsymbol{a_j})]}{\boldsymbol{v_l}\emptyset_{\boldsymbol{l}}(\boldsymbol{a_j})} \le \mathbf{1};\ \boldsymbol{j} = \mathbf{1}, \mathbf{2}, \dots, \boldsymbol{n}$$

$$\boldsymbol{W_k^-} \le \boldsymbol{w_k} \le \boldsymbol{W_k^+};$$

$$\boldsymbol{w_k}, \boldsymbol{v_l} \ge \mathbf{0},\ \forall\ \boldsymbol{k}, \boldsymbol{l}$$

The objective function in (34) presents the efficiency score of DMU_o. Further we use a super efficient model to generate a complete ranking of DMUs/alternatives.

The final LP super efficient CCR model is as follows:

$$\boldsymbol{E_o} = \boldsymbol{Max}\ [\emptyset(\boldsymbol{a_o}) = \textstyle\sum_{k=1}^{q} \boldsymbol{w_k}\emptyset_{\boldsymbol{k}}(\boldsymbol{a_o})]$$

Such that

$$\boldsymbol{v_l}\emptyset_{\boldsymbol{l}}(\boldsymbol{a_o}) = \mathbf{1}; \tag{35}$$

$$\sum_{k=1}^{q} \boldsymbol{w_k}\emptyset_{\boldsymbol{k}}(\boldsymbol{a_j}) - \boldsymbol{v_l}\emptyset_{\boldsymbol{l}}(\boldsymbol{a_j}) \le \mathbf{0};\ \boldsymbol{j} = \mathbf{1}, \mathbf{2}, \dots, \boldsymbol{n}, \qquad \boldsymbol{j} \ne \boldsymbol{o}$$

$$\boldsymbol{W_k^-} \le \boldsymbol{w_k} \le \boldsymbol{W_k^+};$$

$\boldsymbol{w_k, v_l \geq 0, \forall\, k, l}$

In the next section, we will compare the results between different combinations of DEA and MCDA methods. Moreover, we will restrict the stability interval level to the first position.

5. Numerical examples

In this section, we illustrate the application of the proposed model on three different data sets. The first one is based on a research project in water resources management in Greece [36]. This report considers different economic, technical, environmental and social aspects to manage water resources in the agriculture irrigation problem. The aim of the author is to build a set of water pricing policies based on a multicriteria approach. Yilmaz and Yurdusef in 2011 [37] already gave CCR/w, BCC/w and RCCR[18]/w (Reduced CCR/w) rankings based on these data. They applied AR methods to limit DEA weights. The second data set is based on the evaluation of medium-sized enterprises in Brussels. The data are available on the website “Trends Gazelles” [38]. Finally, in third example, we compare the level of well-being in different municipalities of Wallonia (Belgium). Data set are provided from a research report of IWEPS[20] (Institut Wallon de l'Evaluation, de la Prospective et de la Statistique) [39].

Example 1.

This problem includes 36 alternatives and 7 criteria. The alternatives obtained by combination of 4 key factors and their sub factors:

1. Used fertilizers (chemical, green);
2. Water price (very high, high, and moderate)
3. Crop distribution (more fruits/vegetables and less cotton per acre, existing crops) and
4. Different irrigation schemes (drip, sprinkler and surface).

For example, the combination of utilizing chemical fertilizers, moderate water price, existing crop distribution method and surface irrigation scheme factors give the first alternative.

The criteria are classified into two groups:

1. C_1 (crops profitability), C_2 (used water efficiency), C_3 (social impact including employment) that have to be maximized;
2. C_4 (initial cost), C_5 (maintenance cost), C_6 (irrigation water volume used), C_7 (pollution effect) that have to be minimized.

In the DEA analysis the criteria that have to be minimized are considered as inputs (I) and those that have to be maximized as outputs (O). The represented criteria’ weights by DMs are: w_1: 0.3, w_2: 0.25, w_3: 0.09, w_4: 0.1, w_5: 0.1, w_6: 0.1, w_7: 0.06. The data set and details in choosing alternatives, criteria and their weights described in the related report [36]. We consider ranking results of different methods: CCR, BCC, WCCR and the proposed methodology: PROMETHEE II-weighted-CCR (PIIWCCR) and PROMETHEE II-weighted-BCC (PIIWBCC). These results are compared with existing results of Yilmaz and Yurdusef [37] (they presented: CCR/w, BCC/w and RCCR/w). Furthermore we use a Super Efficient (SE[19]) model in CCR, WCCR and PIIWCCR to improve the discrimination power between alternatives scores.

Concerning the MCDA methods we compare two outranking techniques: ELECTRE III (EL.3) and PROMETHEE II (PR.II). In this example the preference threshold of ELECTRE III is fixed to 0.5 and the distillation thresholds are equal to -0.15 and 0.3. ELECTRE III parameterization was fixed based on the work of Yilmaz and Yurdusef [37]. In PROMETHEE, a linear function is chosen for all criteria. The values of the PROMETHEE parameters are listed in Table 1. As already stressed, the unicriterion net flow scores, $\emptyset_k(a_j)$, are used as the output in the DEA analysis. To avoid using negative vales of $\emptyset_k(a_i)$ in DEA problem, the net flow scores are transformed by using a linear transformation: $[\emptyset_k(a_j)+1]/2$.

Criteria	**C_1**	**C_2**	**C_3**	**C_4**	**C_5**	**C_6**	**C_7**
Min/Max	Max	Max	Max	Min	Min	Min	Min
Type	Linear	Linear	Linear	Linear	Linear	Linear	Linear
Thresholds	q=0.1,p=1	q=0,p=0.5	q=0.5,p=1	q=0,p=0.29	q=0.1,p=0.26	q=0,p=0.26	q=0,p=0.46
Weights	0.3	0.25	0.09	0.1	0.1	0.1	0.06

Table 1- PROMETHEE parameters (irrigation management)

Table 2 illustrates PROMETHEE II WSI where the stability level is set to 1.

Criteria	**Min weight**	**Value**	**Max weight**
C_1	0.036	0.3	1
C_2	0	0.25	0.407
C_3	0	0.09	0.529
C_4	0	0.1	0.502
C_5	0	0.1	1
C_6	0	0.1	0.383
C_7	0	0.06	1

Table 2- Weight Stability Intervals of PROMETHEE II in r=1 (irrigation management)

The efficiency scores of the different DMUs are summarized in Table 1 in appendix. The bold numbers show the efficiency scores that are equal to or higher than 1. Obviously, this table presents DMU 26 as the only efficient unit in all mentioned DEA models. In CCR and BCC the number of equal efficient DMUs (Efficiency=1) is respectively 12 and 15. Alternatively, the SE-CCR efficiencies seem a bit different. This confirms suppositions that CCR and BCC alone is not a good discriminator among DMUs. Moreover the SE-CCR model is not enough for a full ranking. As expected, the number of equal efficient units in WCCR, PIIWCCR, PIIWBCC, CCR/w and BCC/w are reduced to 4, 6, 8, 1 and 2 (which is clearly less than CCR and BCC). Though, incorporating weight restrictions in DEA models can be done as a strategy not only to partly integrate the DMs preferences but also to improve the DEA discrimination power by decreasing the number of equal efficient units. Figure 2 illustrates the number of efficient units according to their DEA models:

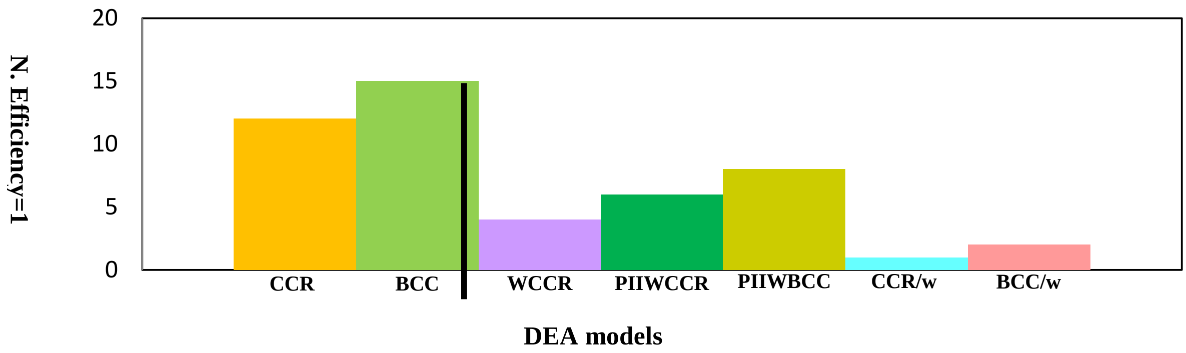

Figure 2-The number of efficient DMUs with efficiency equal to 1 in different DEA methods (Irrigation management)

Table 2 in appendix represents the ranking results of different decision making methodologies. Below, Table 3, outlines the detailed of mentioned table. The rank orders are not exactly similar in different approaches but it can be seen that alternative/DMU 26 is always the most preferred alternative and alternative 11 is among the least preferred ones. The first two alternatives in PROMETHEE II, SE-WCCR and SE-PIIWCCR are exactly the same (alts. 26, 34), however in PIIWBCC, alternative 34 is tied at rank 4. From this table, the DM can determine the best and worst choice by analysing rank order of different methods. Moreover, the rankings induced by different method can be used to determine more robust choices.

Rank	EL.3	PR.II	SE-WCCR	SE-PIIWCCR	PIIWBCC	RCCR/w, CCR/w	BCC/w
1	26	26	26	26	26	26	26
2	28	34	34	34	4	28	30
3	2	30	4	4	28	2	28
.	.	.		.	.	.	.
.	.	.		.	.	.	.
.	.	.		.	.	.	.
34	9	3	23	11	23	21	21
35	11	7	7	19	11	23	11
36	23	11	19	23	9	11	23

Table 3- The best and worst alternatives in different methods (irrigation management)

Table 4 shows the Spearman correlation among the different methods. The correlation values are is significant at the 0.01 level (2-tailed).

	EL.3	**PR.II**	**PIIWCCR**	**PIIWBCC**	**WCCR**	**RCCR/w, CCR/w**	**BCC/w**	**CCR**	**SE-CCR**	**BCC**
EL.3	1	0.608	0.807	0.800	0.823	0.944	0.761	0.678	0.739	0.607
PR.II		1	**0.914**	0.526	0.706	0.652	0.737	0.632	0.625	0.571
PIIWCCR			1	0.896	0.906	0.824	0.877	0.877	0.909	0.898
PIIWBCC				1	0.847	0.769	0.803	0.871	0.912	0.854
WCCR					1	0.826	0.871	0.871	0.923	0.803
RCCR/w, CCR/w						1	0.874	0.683	0.733	0.658

BCC/w	1	0.787	0.806	0.852
CCR		1	0.965	0.895
SE-CCR			1	0.881
BCC				1

Table 4- Spearman r values (correlation is significant at the 0.01 level, 2 tailed) (irrigation management)

According to table 4, incorporating the WSI of PROMETHEE II into DEA as weight constraints seems to provide some results which are correlated to other methods. As expected, there is a high correlation between PIIWCCR and PROMETHEE II.

Example 2.

In the second example we focus ourselves on the ranking of medium-sized companies in Brussels. This is based on the growth value of each criterion during 4 consecutive years (criteria are introduced below). A rank is assigned to each criterion for each company during the years 2008 to 2012. The ranking obtained by "Gazelles" is obtained by adding the rank of each company on each criterion.

Tables 3 and 4 (available in appendix) present the related data of 75 companies according to 6 criteria as well as their efficiency scores for the different models. Revenue (turnover), cash-flow and employees are 3 main criteria that are considered in two groups: Absolute growth and Relative growth. To be included in the category of medium sized companies, their revenue should be between 1 and 10 million Euros in the starting year. In the study, the company has to be registered for at least five years, presented a positive cash flow and engaged at least 20 persons in the last year. More information about data can be found in [38].

This model is a full input case, thus a dummy output with value of 1 for all the DMUs is added to model to solve the CCR model. As Lovell and Pastor explained in [35] an input oriented CCR model with a constant single output corresponds with its BCC model. Thus the number of equal efficient DMUs (efficiency=1) in both CCR and BCC is identical and equal to 7. In PIIWCCR, there are just 3 DMUs with equal efficiency. In the super efficient model, a complete ranking is observed. The results show that DMUs 1, 14 and 37 as the best efficient units in all mentioned DEA models. The bold numbers in table 4 (appendix) show the efficiency scores equal to 1.

We compare the Gazelles rankings with PROMETHEE II, SE-CCR, BCC and PIIWCCR. The preference functions are all linear type and weights are equally distributed. Tables 5 and 6 display PROMETHEE II parameters and the corresponding stability intervals in level 1 (r=1).

Criteria	**C_1**	**C_2**	**C_3**	**C_4**	**C_5**	**C_6**
Min/Max	Min	Min	Min	Min	Min	Min
Type	Linear	Linear	Linear	Linear	Linear	Linear
Thresholds	q=20,p=80	q=20,p=85	q=25,p=90	q=30,p=95	q=20,p=75	q=20,p=75
Weights	0.167	0.167	0.167	0.167	0.167	0.167

Table 5- PROMETHEE parameters (Gazelles)

Criteria	**Min weight**	**Value**	**Max weight**
C_1	0	0.167	1

C_2	0	0.167	1
C_3	0	0.167	0.899
C_4	0	0.167	0.706
C_5	0	0.167	1
C_6	0	0.167	0.986

Table 6- Weight Stability Intervals of PROMETHEE II in r=1 (Gazelles)

Table 5 in appendix represents the ranking results of the considered methodologies. Let us point out that the company "Hennes & Mauritz Logistic" has rank 1, in all methods. This pleads in favor of a robust conclusion. The first two companies in Gazelles and PROMETHEE II are the same (second company is Pull & Bear) however in PIIWCCR, company number 37, BCC Corporate, is in the second place. The DM can determine the best company by analysing rank order of different methods in this table. Table 7 presents the Spearman rank correlation between rank results of different methods.

	SE-PIIWCCR	**PR.II**	**SE-CCR**	**BCC**	**Gazelles**
SE-PIIWCCR	1	0.784	0.857	0.860	0.807
PR.II		1	0.622	0.623	0.991
SE-CCR			1	0.989	0.641
BCC				1	0.645
Gazelles					1

Table 7- Spearman r values (correlation is significant at the 0.01 level, 2 tailed) (Gazelles)

First of all, let us point out that there is a high correlation value between PROMETHEE II and Gazelles. This confirms the fact that the chosen parameters (for PROMETHEE II) are compatible with the initial ranking (Gazelles). At a level of stability equal to 1, generated intervals (table 6) are not tight enough to have a clear impact in the new method. Thus, the correlation between SE-PIIWCCR and basic DEA models (SE-CCR, BCC) is still more than related correlation between SE-PIIWCCR and PROMETHEE II. It leads us to go to another level of stability in PROMETHEE II to generate more restricted intervals to address this problem. As it has been mentioned on the Gazelles website [38], **Moore Stephens** is the winner among the group of medium-sized companies in Brussels and has the leadership role. Table 5 in appendix shows Moore Stephens has 3rd place in PROMETHEE II ranking. Therefore, we fix the level of stability to 3. Obviously when the WSI is considered at level 3 (r=3), the ranges are likely to be reduced, there will be less equal efficient DMUs and more correlation between rankings. Next tables show WSI in r=3 and the existed correlation among methodologies.

Criteria	**Min weight**	**Value**	**Max weight**
C_1	0.14	0.167	0.246
C_2	0.058	0.167	0.319
C_3	0.047	0.167	0.203
C_4	0.131	0.167	0.204
C_5	0	0.167	0.317
C_6	0.096	0.167	0.323

Table 8- Weight Stability Intervals of PROMETHEE II in r=3 (Gazelles)

Table 6 in appendix represents the rankings of PIIWCCR and PIIWBCC when stability level is changed to 3. As it is clear company number 1, Hennes & Mauritz Logistic, is always stays in the first rank. Moreover, the number of companies with equal efficiency is reduced to just 1 (we avoid presenting table of efficiency scores).

	PIIWCCR	PIIWBCC	PR.II	CCR	BCC	Gazelles
PIIWCCR	1	0.906	0.962	0.643	0.650	0.915
PIIWBCC		1	0.961	0.695	0.702	0.958
PR.II			1	0.619	0.626	0.981
CCR				1	0.998	0.639
BCC					1	0.645
Gazelles						1

Table 9- Spearman r values (correlation is significant at the 0.01 level, 2 tailed), the level of stability interval=3 (Gazelles)

Example 3.

In this example, we compare the level of well-being in different municipalities of Wallonia (Belgium). Data are available in [39]. They evaluated 262 municipalities according to 20 different well-being indices. Here we decrease the number of municipalities to 132 (trying to evaluate the most important ones) and criteria to 13. The criteria are health-care, accommodation, education and training, employment, income and purchasing power, mobility, quality of life and environment, proximity to shopping centres, security of life and environment, administrative institutions, the situation of marital and family, feeling happy or unhappy and the revenue of municipality. We minimize the last criterion (revenue of municipality) and maximize others. Tables 10 and 11 give details of the PROMETHEE II parameters and the related stability intervals at level 1. Finally, let us stress that the weights are equally chosen.

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7
Min/Max	Max	Max	Max	Max	Max	Max	Max
Type	Linear	Linear	Linear	Linear	Linear	Linear	Linear
Thresholds	q=0.1,p=0.6	q=0.05,p=0.4	q=0.01,p=0.5	q=0.05,p=0.4	q=0.02,p=0.45	q=0,p=0.5	q=0.01,p=0.45
Criteria	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	
Min/Max	Max	Max	Max	Max	Max	Min	
Type	Linear	Linear	Linear	Linear	Linear	Linear	
Thresholds	q=0,p=0.5	q=0,p=0.5	q=0,p=0.5	q=0,p=0.4	q=0.1,p=0.5	q=2000,p=8000	

Table 10- PROMETHEE parameters (weights are equal) (well-being level in Wallonia)

Criteria	Min weight	Value	Max weight
C_1	0	0.077	0.112
C_2	0.061	0.077	0.334
C_3	0.037	0.077	0.174
C_4	0	0.077	0.476
C_5	0.02	0.077	0.414
C_6	0	0.077	0.095
C_7	0.044	0.077	0.691
C_8	0	0.077	0.104
C_9	0	0.077	0.547

C_{10}	0.055	0.077	1
C_{11}	0	0.077	0.108
C_{12}	0	0.077	0.243
C_{13}	0	0.077	0.505

Table 11- Stability intervals in level 1 (well-being level in Wallonia)

As in the two previous examples, the integration of weight intervals to DEA model allows to decrease the number of efficient units. These numbers in CCR and BCC models are respectively equal to 42 and 85 whereas in WCCR, PIIWCCR and PIIWBCC they become 21, 25 and 37. Figure 3 illustrates this comparison. Table 7 in appendix displays the ranking results of SE-CCR, SE-WCCR, PROMETHEE II, SE-PIIWCCR and PIIWBCC. It is noticeable that the municipalities 118 and 92, Tintigny and Ottignies-LLN, are most of the time in the top of the different ranking methods.

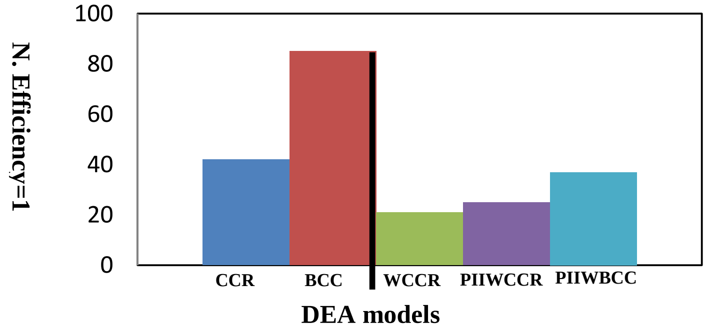

Figure 3-The number of efficient DMUs with efficiency equal to 1 in different DEA methods (well-being level in Wallonia)

The main point that can be seen in table 12 is that the basic DEA models, SE-CCR and BCC, have very low correlation* with PROMETHEE II (0.136, 0.281) while the correlation** between SE-PIIWCCR and PIIWBCC with PROMETHEE II are quite high (0.881, 0.81).

	SE-PIIWCCR	PIIWBCC	SE-WCCR	PR.II	SE-CCR	BCC
SE-PIIWCCR	1	0.844	0.394	0.881**	0.376	0.486
PIIWBCC		1	0.444	0.810**	0.51	0.529
SE-WCCR			1	0.182	0.915	0.446
PR.II				1	0.136*	0.281*
SE-CCR					1	0.500
BCC						1

Table 12-Spearman r values (correlation is significant at the 0.01 level, 2 tailed) (well-being level in Wallonia)

6. Conclusion

In this work, we have investigated the potential synergies between DEA and a particular multicriteria method, PROMETHEE. The underlying idea is to use the outputs of the sensitivity analysis of PROMETHEE, namely the weight stability intervals, in order to reduce the weights' degrees of freedom in a DEA analysis. The expected benefits are two folds:

1) On the one hand to reduce the number of DMUs that are characterized by an efficient score equal to 1;
2) On the other hand, to partly integrate the preferences of the decision maker in a DEA analysis (and to obtain outputs that are compatible with a MCDA ranking);

The proposed method is based on a DEA approach based on the unicriterion net flow scores. This has been applied on 3 real case studies. A comparison has been done with different DEA and MCDA methods. Clearly, the number of efficient DMUs is decreased in weighted DEA models. As expected, there exist a considerable correlation between the proposed approach, MCDA methods and basic DEA models. Finally, weights intervals are determined automatically (depending on the acceptation of the PROMETHEE ranking and given a certain stability level). In other words, if the DM is confident with the PROMETHEE ranking, he may rely on these computed weights intervals and does not have to fix these values himself.

In future works, the stability intervals can be applied in the proportional form in DEA. Using partial or subset of stability intervals of PROMETHEE can be discussed in DEA models, too. Furthermore, the likelihood of infeasibility has to be further analyzed (even it has not been observed in the three case studies).

To the best of our knowledge, this is a first work in which one investigates the potential synergies between PROMETHEE and DEA. In this first attempt, we decided to use the tools from MCDA to apply it on DEA. To conclude this paper, we want to stress that the question can be addressed in the other way. For instance, how the outputs of DEA (i.e. a ranking with numerous ties) can be used in the PROMETHEE VI procedure. This will be the main topic of a forthcoming contribution by the same authors.

Appendix

DMUs/Alts	**CCR**	**BCC**	**SE-CCR**	**SE-WCCR**	**SE-PIIW CCR**	**PIIW BCC**	**CCR/w**	**BCC/w**	**RCCR/w**
1	0.857	0.939	0.857	0.814	0.9031	0.989	0.745	0.856	0.745
2	**1**	**1**	**1.016**	0.994	**1.0269**	**1**	0.904	0.946	0.904
3	0.845	0.905	0.845	0.829	0.8771	0.990	0.711	0.825	0.712
4	**1**	**1**	**1.159**	**1.054**	**1.0630**	**1**	0.855	0.902	0.855
5	0.792	0.939	0.792	0.749	0.8982	0.986	0.650	0.856	0.651
6	0.974	**1**	0.974	0.856	0.9943	0.996	0.796	0.946	0.796
7	0.779	0.905	0.779	0.710	0.8631	0.987	0.617	0.825	0.617
8	0.960	**1**	0.960	0.852	0.9622	0.996	0.752	0.902	0.752
9	0.825	0.915	0.825	0.790	0.8136	0.983	0.598	0.836	0.598
10	**1**	**1**	**1.008**	0.896	0.9424	**1**	0.732	0.921	0.733
11	0.819	0.900	0.819	0.739	0.7857	0.983	0.562	0.801	0.563
12	**1**	**1**	**1.019**	0.860	0.9418	**1**	0.686	0.873	0.687
13	0.826	0.909	0.826	0.757	0.8775	0.992	0.737	0.838	0.738
14	0.922	0.950	0.922	0.921	0.9369	0.996	0.902	0.936	0.903
15	0.772	0.847	0.772	0.752	0.8084	0.989	0.715	0.821	0.715
16	0.944	0.958	0.944	0.942	0.9631	0.996	0.870	0.911	0.871
17	0.826	0.909	0.826	0.740	0.8744	0.989	0.654	0.838	0.654
18	0.891	0.950	0.891	0.866	0.9223	0.992	0.803	0.936	0.804
19	0.705	0.847	0.705	0.692	0.7674	0.986	0.624	0.821	0.625
20	0.886	0.958	0.886	0.824	0.9370	0.993	0.767	0.911	0.767
21	0.833	0.909	0.833	0.771	0.7949	0.986	0.598	0.814	0.598

22	0.913	0.952	0.913	0.899	0.9240	0.989	0.739	0.910	0.739
23	0.738	0.852	0.738	0.730	0.7518	0.983	0.570	0.796	0.570
24	0.919	0.958	0.919	0.882	0.9398	0.990	0.697	0.876	0.697
25	**1**	**1**	**1**	0.871	0.9963	0.996	0.831	0.900	0.831
26	**1**	**1**	**1.117**	**1.071**	**1.1310**	**1**	**1**	**1**	**1.055**
27	0.848	0.909	0.848	0.828	0.8819	0.992	0.804	0.887	0.805
28	**1**	**1**	**1.084**	**1.051**	**1.0519**	**1**	0.975	0.983	0.976
29	**1**	**1**	**1**	0.853	0.9929	0.992	0.741	0.900	0.741
30	**1**	**1**	**1**	0.985	0.9963	0.996	0.900	**1**	0.900
31	0.826	0.909	0.826	0.791	0.8744	0.989	0.712	0.887	0.712
32	0.978	**1**	0.978	0.930	0.9966	0.996	0.865	0.983	0.865
33	**1**	**1**	**1**	0.893	0.9608	0.996	0.682	0.877	0.682
34	**1**	**1**	**1.134**	**1.069**	**1.0644**	**1**	0.836	0.978	0.836
35	0.833	0.909	0.833	0.820	0.8493	0.986	0.654	0.864	0.655
36	**1**	**1**	**1.029**	0.987	**1.0229**	**1**	0.796	0.953	0.796

Table 1- The efficiency scores of DEA models (irrigation management)

Rank	**MCDA models**		**Weighted DEA models**				
	EL.3	**PR.II**	**SE-WCCR**	**SE-PIIWCCR**	**PIIWBCC**	**RCCR/w, CCR/w**	**BCC/w**
1	26	26	26	26	26	26	26
2	28	34	34	34	4	28	30
3	2	30	4	4	28	2	28
4	4	25	28	28	34	14	32
5	14	28	2	2	2	30	34
6	16	33	36	36	36	16	36
7	25	36	30	32	12	32	2
8	30	14	16	30	10	4	6
9	32	29	32	25	8	34	14
10	27	22	14	6	6	25	18
11	34	32	22	29	33	27	10
12	36	18	10	16	16	18	16
13	3	35	33	8	32	36	20
14	8	27	24	33	25	6	22
15	18	10	25	10	30	20	8
16	33	2	18	12	14	8	4
17	1	31	12	24	20	1	29
18	13	13	6	20	18	29	25
19	6	16	29	14	29	22	31
20	29	24	8	22	27	13	27
21	15	6	3	18	13	10	33
22	20	17	27	1	3	15	24
23	22	21	20	5	24	31	12
24	10	20	35	27	15	3	35
25	24	1	1	13	31	24	5
26	31	9	31	3	22	12	1
27	12	4	9	31	17	33	17
28	35	15	21	17	1	35	13
29	17	5	13	7	7	17	9
30	5	23	15	35	35	5	7
31	7	12	5	9	19	19	3
32	21	19	17	15	21	7	15
33	19	8	11	21	5	9	19
34	9	3	23	11	23	21	21

35	11	7	7	19	11	23	11
36	23	11	19	23	9	11	23

Table 2- Increasing rank order of alternatives (irrigation management)

	Absolute Growth of Revenue	Relative Growth of Revenue %	Absolute Growth of Cash-flow	Relative Growth of Cash-flow %	Absolute Growth of Employment	Relative Growth of Employment%
1 H & M Logist.	3	1	5	7	1	2
2 Pull&Bear	13	9	11	3	7	9
3 MCA Benelux	12	7	10	24	2	8
4 M. Stephens	**5**	**5**	**16**	**30**	**3**	**5**
5 Jules	9	3	28	23	5	4
6 Codasoft	30	6	23	4	13	14
7 Cognizant Tec.	6	8	32	28	24	13
8 C. Tour&Fin	20	13	27	16	22	19
9 Aremis B.	35	15	44	15	9	7
10 Histoire d'Or	18	10	21	40	15	23
11 BuSI	4	11	17	21	39	42
12 Claire's	16	17	15	42	8	38
13 Robert Ronny	31	24	7	46	23	10
14 Lubrizol	8	14	2	80	11	32
15 Bezoom	28	29	3	27	16	47
16 Google	7	12	24	26	63	50
17 Fleishman-H.	19	35	30	49	20	35
18 E.S. Network	10	26	57	113	4	24
19 Inter-Invest	49	44	60	5	43	36
20 Tastyfood	22	48	77	8	32	52
21 Be films	44	60	80	38	19	6
22 Watteau R.	11	20	70	96	44	11
23 Axemedia	84	22	85	10	42	27
24 Flanders' D.	34	16	50	118	38	22
25 Caviar	17	33	63	90	58	28
26 Beijaflore	69	40	69	29	49	40
27 Xperthis	23	25	6	62	103	90
28 Troostwijk	68	36	76	66	61	25
29 DST	79	32	84	85	26	31
30 Thalys Int.	15	18	81	111	33	79
31 Numerical M.	41	50	52	117	28	51
32 Marsh Europe	93	96	34	51	56	15
33 RSM InterFid.	56	41	108	87	34	20
34 Eur. Amuse.	61	38	86	107	29	29
35 Matis Ben.	25	43	120	94	12	56
36 Grayling	71	28	92	101	51	17
37 BCC Corp.	26	57	8	1	138	130
38 Securitas Dir.	88	45	134	100	10	1
39 Misanet	55	51	118	55	21	78
40 Edital	51	54	42	97	81	61
41 Macq	43	53	83	115	36	58
42 3E	52	71	106	124	14	30
43 Zara Home	57	63	33	18	114	112
44 Baguette M.B	29	39	43	56	125	123
45 Verano Be	42	80	73	67	68	86
46 M.C.A	96	55	47	79	78	65
47 ISS Europe	54	78	12	50	107	119
48 Comax	80	108	53	75	48	59

49 Ocean Maree	36	70	56	84	98	80
50 Potiez-Deman	117	109	99	59	18	26
51 Lambiotte&Ci	33	59	18	34	154	155
52 KPMG Fid.	47	76	91	83	60	99
53 N. Sobelwood	46	91	101	138	31	69
54 Kiala	108	31	148	134	40	18
55 ATF	72	102	54	82	80	100
56 Sonauto	74	66	46	72	126	106
57 SMB pharma	120	99	122	57	50	43
58 Mediaplanet	82	23	159	168	54	12
59 RSM Interau.	40	34	189	217	17	3
60 Caloribel	64	100	31	103	85	117
61 VGD	97	89	79	112	66	60
62 Bedimo	39	58	110	120	93	88
63 Fiducial	32	67	146	150	52	62
64 Longchamp	48	47	132	145	77	68
65 Recticel Inter.	76	21	210	157	41	16
66 Sopra Belux	91	128	64	33	89	116
67 Interfashion	122	68	141	6	102	84
68 Ralph Lauren	113	90	95	114	55	57
69 Calidat	144	118	93	71	57	53
70 Carodec	66	77	119	109	94	77
71 Senec	81	114	67	35	109	143
72 Denis-Buxant	160	175	45	60	37	75
73 IKKS	38	56	221	186	25	33
74 VDS-food	21	46	207	180	62	54
75 The Swatch g.	83	119	48	58	118	144

Table 3- Data set of medium-sized companies (Gazelles)

DMUs	**CCR**	**BCC**	**PIIW CCR**	***DMUs***	**CCR**	**BCC**	**PIIW CCR**
1	**1**	**1**	**1**	**47**	0.335	0.335	0.912
2	**1**	**1**	0.995	**48**	0.094	0.094	0.756
3	0.500	0.500	0.989	**49**	0.088	0.088	0.795
4	0.600	0.600	0.987	**50**	0.110	0.110	0.896
5	0.497	0.497	0.988	**51**	0.266	0.266	0.918
6	**1**	**1**	0.992	**52**	0.081	0.080	0.713
7	0.500	0.500	0.974	**53**	0.065	0.065	0.814
8	0.341	0.341	0.946	**54**	0.108	0.108	0.892
9	0.428	0.428	0.969	**55**	0.092	0.091	0.716
10	0.231	0.231	0.940	**56**	0.107	0.107	0.764
11	0.750	0.750	0.979	**57**	0.099	0.099	0.769
12	0.312	0.312	0.957	**58**	0.161	0.161	0.925
13	0.650	0.650	0.978	**59**	0.586	0.586	0.970
14	**1**	**1**	**1**	**60**	0.142	0.142	0.796
15	**1**	**1**	0.979	**61**	0.063	0.063	0.633
16	0.428	0.428	0.963	**61**	0.076	0.076	0.761
17	0.165	0.165	0.900	**63**	0.093	0.093	0.798
18	0.300	0.300	0.974	**64**	0.062	0.062	0.741
19	0.562	0.562	0.918	**65**	0.122	0.122	0.913
20	0.488	0.488	0.944	**66**	0.133	0.133	0.796
21	0.323	0.323	0.954	**67**	0.426	0.426	0.814
22	0.272	0.272	0.948	**68**	0.058	0.058	0.675
23	0.352	0.352	0.937	**69**	0.080	0.080	0.694
24	0.099	0.099	0.908	**70**	0.060	0.060	0.583

25	0.176	0.176	0.900	**71**	0.126	0.126	0.783
26	0.157	0.157	0.848	**72**	0.113	0.113	0.808
27	0.572	0.572	0.958	**73**	0.078	0.078	0.861
28	0.101	0.101	0.861	**74**	0.142	0.142	0.853
29	0.079	0.079	0.861	**75**	0.111	0.110	0.783
30	0.200	0.200	0.919				
31	0.092	0.092	0.836				
32	0.146	0.146	0.911				
33	0.099	0.099	0.881				
34	0.068	0.068	0.856				
35	0.120	0.120	0.918				
36	0.115	0.115	0.900				
37	**1**	**1**	**1**				
38	**1**	**1**	0.989				
39	0.111	0.111	0.858				
40	0.112	0.112	0.790				
41	0.069	0.069	0.799				
42	0.071	0.071	0.905				
43	0.241	0.241	0.908				
44	0.121	0.121	0.843				
45	0.097	0.097	0.757				
46	0.103	0.103	0.765				

Table 4- The efficiency scores of DEA models when r=1 in determining weight stability intervals in PROMETHEE II (Gazelles)

Rank	GAZ.	PR.II	SE-CCR	BCC	PIIW CCR	Rank	GAZ.	PR. II	SE-CCR	BCC	PIIW CCR
1	**1**	**1**	**1**	**1**	**1**	**39**	39	38	66	66	17
2	**2**	**2**	**37**	**37**	**37**	**40**	40	42	71	71	50
3	3	**4(M.S)**	38	38	14	**41**	41	40	65	65	54
4	4	3	14	14	2	**42**	42	37	44	44	33
5	5	5	2	2	6	**43**	43	45	35	35	73
6	6	6	15	15	3	**44**	44	48	36	36	28
7	7	7	6	6	38	**45**	45	43	72	72	29
8	8	8	11	11	5	**46**	46	59	40	40	39
9	9	9	13	13	4	**47**	47	44	39	39	34
10	10	10	4	4	15	**48**	48	46	75	75	74
11	11	11	59	59	11	**49**	49	49	50	50	26
12	12	12	27	27	13	**50**	50	50	54	54	44
13	13	13	19	19	18	**51**	51	51	56	56	31
14	14	14	7	3	7	**52**	52	47	46	46	67
15	15	15	3	7	59	**53**	53	52	28	28	53
16	16	17	5	5	9	**54**	54	65	57	57	72
17	17	16	20	20	16	**55**	55	53	24	24	41
18	18	18	9	9	27	**56**	56	54	33	33	63
19	19	19	16	16	12	**57**	57	73	45	45	60
20	20	20	67	67	21	**58**	58	58	48	48	66
21	21	21	23	23	22	**59**	59	63	63	63	49
22	22	22	8	8	8	**60**	60	57	31	31	40
23	23	24	47	47	20	**61**	61	74	55	55	75
24	24	23	21	21	10	**62**	62	64	49	49	71
25	25	25	12	12	23	**63**	63	56	52	52	57
26	26	26	18	18	58	**64**	64	61	69	69	46
27	27	31	22	22	30	**65**	65	55	29	29	56
28	28	28	51	51	35	**66**	66	62	73	73	62

29	29	29	43	43	51	**67**	67	72	62	62	45
30	30	33	10	10	19	**68**	68	60	42	42	48
31	31	34	30	30	65	**69**	69	68	41	41	64
32	32	30	25	25	47	**70**	70	69	34	34	55
33	33	35	17	17	32	**71**	71	70	53	53	52
34	34	27	58	58	24	**72**	72	66	61	61	69
35	35	36	26	26	43	**73**	73	67	64	64	68
36	36	32	32	32	42	**74**	74	71	70	70	61
37	37	41	74	74	25	**75**	75	75	68	68	70
38	38	39	60	60	36						

Table 5- Ranking results, (M.S.: Moore Stephens, the leader company between medium-sized companies in Brussels) (Gazelles)

Rank	PIIWCCR	PIIW BCC	Rank	PIIWCCR	PIIWBCC
1	**1**	**1**	**39**	65	31
2	4	2	**40**	27	30
3	5	4	**41**	73	40
4	2	5	**42**	58	49
5	3	3	**43**	39	39
6	6	7	**44**	54	45
7	9	6	**45**	41	47
8	7	8	**46**	50	41
9	8	11	**47**	37	42
10	10	9	**48**	40	38
11	14	10	**49**	74	59
12	12	13	**50**	48	48
13	13	16	**51**	44	52
14	11	12	**52**	45	46
15	15	15	**53**	63	50
16	18	14	**54**	43	56
17	17	17	**55**	51	62
18	16	19	**56**	46	73
19	22	20	**57**	49	65
20	21	21	**58**	52	63
21	24	22	**59**	53	74
22	23	25	**60**	57	58
23	20	18	**61**	47	64
24	19	23	**62**	64	67
25	25	37	**63**	72	57
26	59	26	**64**	69	55
27	38	24	**65**	56	53
28	33	27	**66**	62	71
29	29	28	**67**	61	54
30	34	33	**68**	68	70
31	26	51	**69**	55	66
32	35	44	**70**	60	60
33	36	32	**71**	67	75
34	30	43	**72**	70	61
35	28	35	**73**	66	69
36	31	36	**74**	71	68
37	32	34	**75**	75	72
38	42	29			

Table 6- Ranking result while r=3 (Gazelles)

Rank	**SE-CCR**	**PR. II**	**SE-PIIW CCR**	**PIIW BCC**	**SE-WCCR**	**Rank**	**SE-CCR**	**PR. II**	**SE-PIIW CCR**	**PIIW BCC**	**SE-WCCR**
1	**118**	**118**	**118**	**118**	**92**	**71**	38	90	6	32	39
2	**92**	**92**	**92**	21	44	**72**	2	130	43	97	45
3	24	22	120	**92**	11	**73**	83	99	90	46	74
4	108	55	21	120	108	**74**	13	30	63	103	34
5	22	120	91	91	22	**75**	132	124	61	115	50
6	44	132	22	22	100	**76**	74	102	52	76	114
7	11	85	42	42	105	**77**	80	94	14	110	6
8	105	75	55	11	76	**78**	111	61	113	30	59
9	1	7	11	55	62	**79**	23	73	102	65	111
10	76	84	72	108	99	**80**	54	116	73	125	82
11	100	106	106	72	120	**81**	94	43	94	1	46
12	56	11	84	106	79	**82**	50	86	70	27	109
13	120	100	108	66	**118**	**83**	109	111	10	10	102
14	7	66	66	84	60	**84**	46	129	124	14	81
15	62	42	101	34	32	**85**	102	103	130	36	115
16	79	78	79	79	24	**86**	121	74	37	26	83
17	91	4	67	101	7	**87**	82	125	98	107	124
18	106	28	7	54	106	**88**	33	96	57	111	10
19	16	54	34	105	37	**89**	45	18	121	70	85
20	60	91	100	85	58	**90**	39	128	96	61	97
21	19	51	48	67	25	**91**	97	121	111	75	70
22	32	34	54	100	131	**92**	34	80	74	80	128
23	131	21	68	48	69	**93**	96	71	15	53	54
24	90	101	60	75	1	**94**	114	10	16	81	113
25	99	68	19	7	12	**95**	85	112	23	63	29
26	70	88	75	83	101	**96**	47	1	47	69	47
27	21	39	78	41	67	**97**	128	5	109	128	63
28	37	67	85	68	16	**98**	122	40	86	64	55
29	58	8	132	19	19	**99**	59	95	36	47	122
30	72	83	131	60	72	**100**	55	15	44	116	112
31	101	97	105	28	103	**101**	125	47	38	102	51
32	69	41	4	35	91	**102**	64	93	13	130	43
33	28	108	129	132	31	**103**	89	2	69	93	33
34	25	19	99	49	21	**104**	129	113	30	2	64
35	40	17	20	131	8	**105**	51	109	18	57	18
36	42	77	31	8	3	**106**	65	38	29	109	14
37	49	6	115	56	77	**107**	9	13	128	38	107
38	66	114	35	20	84	**108**	63	36	32	43	57
39	8	89	83	78	15	**109**	119	119	95	123	93
40	124	63	17	117	78	**110**	43	9	125	96	129
41	12	48	45	4	117	**111**	107	126	93	74	88
42	86	31	49	77	48	**112**	14	29	62	82	52
43	81	117	1	114	123	**113**	87	23	112	3	9
44	67	105	28	89	116	**114**	112	16	26	13	89
45	3	79	58	31	110	**115**	88	70	81	126	96
46	31	35	56	88	90	**116**	93	3	24	25	4
47	116	127	59	118	75	**117**	68	69	71	95	125
48	77	12	8	58	28	**118**	5	32	2	15	65
49	110	56	122	127	40	**119**	57	62	123	52	95
50	103	122	39	44	80	**120**	126	82	3	29	68
51	113	60	110	87	20	**121**	53	81	5	37	126
52	20	131	51	129	132	**122**	18	107	107	73	130

53	36	57	87	51	2	**123**	52	104	76	5	71
54	15	65	114	113	56	**124**	17	64	126	121	119
55	35	45	77	17	35	**125**	4	37	119	71	5
56	10	72	27	90	13	**126**	95	123	25	119	53
57	30	20	89	86	36	**127**	71	76	82	18	87
58	123	87	117	122	49	**128**	130	44	9	9	27
59	6	27	97	16	73	**129**	27	24	33	23	17
60	84	110	116	124	121	**130**	26	33	50	33	26
61	115	58	88	6	30	**131**	98	50	64	50	98
62	48	49	46	99	66	**132**	104	25	104	104	104
63	41	46	127	59	86						
64	29	59	80	62	42						
65	75	14	103	24	41						
66	127	52	65	40	23						
67	117	98	41	12	61						
68	61	26	40	45	94						
69	73	115	12	94	38						
70	78	53	53	39	127						

Table 7- Ranking results (well-being level in Wallonia)

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