Electromagnetic wave propagation prediction using spatial statistics: experimental validation

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Abstract - The spatial statistics formalism is adapted to electromagnetic wave propagation analysis. Field patterns are considered as realizations of random functions. Their spatial structure is studied thanks to a method known as variographic analysis. To infer unknown values of the fields, an interpolation method called kriging is then applied. It is shown how kriging can be performed on experimental data to speed up the wave propagation prediction process.

1 Introduction

To predict electromagnetic field strength in indoor or outdoor environment, engineers have two possibilities: on the one hand to use numerical codes, and on the other hand to perform a measurement campaign. When the area of interest is large compared to the signal wavelength, both methods are very time-consuming. Ray-tracing algorithms are heavy to use when including multiple effects, and since these algorithms do not give any information about the fields between the computation points, accuracy requirement generally imposes several computations per wavelength, drastically increasing the computation time. In the same way, obtaining extended experimental results over large areas requires an important work and is impractical as a generalized method. Both approaches are thus limited by the dense sampling grid necessary to describe fields, even when only general field characteristics are looked for, as it is often the case. To speed up the electromagnetic wave propagation prediction, a solution would be to deduce those characteristics thanks to only few samples, easy and fast to compute or to measure.

Basically, the problem faced with ray-tracing codes or experimentation can be formulated as follows: given a set of electric field values in a given area and eventually distant from each others by several wavelengths, what can be deduced about the general signal strength distribution throughout the whole area ? Spatial statistics and especially the method known as kriging provide an answer to this important question.

Before being applied to biology or image processing, spatial statistics has been historically developed in the framework of geostatistics [1] to provide mineral grade estimations for mine exploitation. In this specific case, the mineral content distribution was considered as a random function (RF) of position, and thanks to a few mineral samples and a structural tool known as the variogram, the estimated value of this RF was inferred at various locations throughout the mine. The basic idea of the method we propose is to apply this technique to electromagnetic wave propagation prediction: the electric field values within a given area can be considered as one realization of a RF. Knowing samples of this RF thanks to ray-tracing or measurements, it could be possible thanks to spatial statistics to infer the RF values in the whole area. Of course, due to the highly oscillating behaviour of fields, it is impossible to reproduce their exact fast variations, and spatial statistics inference will rather provide electric field envelope estimations and, more important, their estimation variance, i.e. their confidence interval.

2 Spatial Statistics

It is well-known that electromagnetic field patterns in propagation problems are very irregular. They cannot be described by a given explicit mathematical expression and a probabilistic approach must rather be chosen. Field patterns however present spatial structure: the field values are spatially correlated. To explicitly take into account these two properties (stochastic nature and spatial correlation), each field value can be considered as the outcome of some underlying random process and a field pattern can

be viewed as one realization of a random function $Z(x)$ having some statistical properties. To perform inference on the process using this single realization, it is necessary to make the ergodicity assumption as usual in propagation problems. Moreover, as will become clear later on, to apply spatial statistics tools like kriging, the mean and the covariance of the RF $Z(x)$ have to be stationary over the region of interest to be meaningful. In this case, the RF is said to be *second-order stationary*.

The spatial structure of the RF realization can be studied by looking at the associated RF $Z(x)$ moments. By restricting ourselves to second order stationary RF we have chosen to focus only on the first two moments of $Z(x)$. Its first moment (the mathematical expectation) is under second-order stationarity hypothesis a constant and it does not give any information about the spatial structure. On the other hand, the second moment allows to study the similarity or dissimilarity between pairs of values as a function of their spatial lag. Moreover, this second moment will make possible to infer unknown values of the realization thanks to kriging. In spatial statistics, the second moment study is called the variographic analysis.

The variogram $\gamma(h)$ measures the dissimilarity between values of $Z(x)$ at x and $x+h$:

$$\gamma(h) \;=\; \frac{1}{2}\,\mathrm{Var}\{Z(x+h) \;-\; Z(x)\}$$

where Var$\{\}$ indicates the variance of a random variable. It is important to note that, in general, x and h are vectors, so that $\gamma(h)$ can depend on both distance and direction. The variogram is a powerful tool to study the spatial structure of experimental or numerical data. It allows to detect inhomogeneities, anomalies or eventual anisotropy in the multi-dimensional case. The general behaviour of $\gamma(h)$ is drawn on Fig.1 in the uni-dimensional case. The variogram represents the average square increment between pairs of values separated by a spatial lag h, so that it generally increases with distance. The variogram behaviour near the origin indicates the data regularity. If $\gamma(0) \neq 0$, the variogram is said to present a *nugget effect* reflecting either a physical phenomenon at smaller scale which has not been well resolved by the experiment or computation or an experimental or computational error too important compared to the regionalized variable scale. At a distance called the *range*, the variogram reaches a *sill* indicating the absence of spatial structure i.e. the absence of correlation. The sill level is a measure of the variation amplitude of the data.

To each physical process underlying the RF corresponds an experimental variogram, and $\hat{\gamma}(h)$ can be viewed as the common signature of regionalized variables sharing the same origin. In wave propagation, different experimental variograms will be found for different kinds of propagation: log-normal shadowing, Rayleigh or Rice fast fading,.. It is possible on physical grounds to theoretically derive its mathematical expression [2]. But the variogram is not just a diagnostic tool useful to explore experimental or numerical data, it also plays a major role in unknown values inference thanks to kriging.

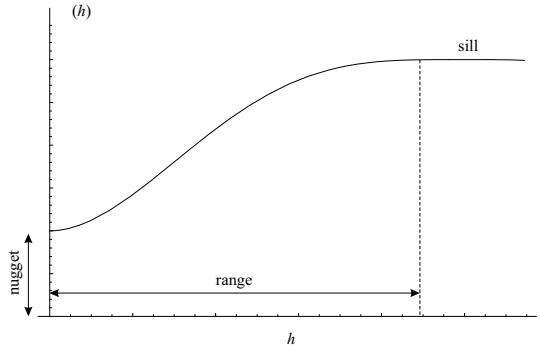

Fig.1. Typical variogram

Let us now consider a set of field values obtained numerically or experimentally. To be able to deduce useful information like unknown field values, from these few data, it would be necessary to know the local spatial structure of the global field pattern. This structure can be given by the theoretical variogram $\hat{\gamma}(h)$. The process of inference taking into account spatial structure thanks to the variogram is called *kriging*.

The kriging theory is defined in the probabilistic framework of field patterns. Let $\hat{Z}(x_0)$ be an estimation of the unknown variable $Z(x_0)$. For this estimation to be linear in the known points, it is written as a sum of n variables $Z(x_\alpha)$ in the x_0 neighborhood:

$$\hat{Z}(x_0) \;=\; \sum_{\alpha=1}^{n} w_\alpha\, Z(x_\alpha)$$

where w_α are coefficients to be determined. Generally, to decrease the computation time, kriging is applied in the close neighborhood of x_0, i.e. n is usually chosen very low.

This estimator is imposed to be unbiased and to be optimal in the sense that it minimizes the estimation error variance

$$\sigma_e^2(x_0) \;=\; \mathrm{Var}\{\hat{Z}(x_0) - Z(x_0)\} \qquad (1)$$

By minimizing $\sigma_e^2(x_0)$ with the unbiasness constraint introduced using a Lagrange multiplier μ, a kriging system

for the w_α and μ coefficients is obtained [1]:

$$\begin{cases} \sum_{\beta=1}^{n} w_\beta \, \gamma(|x_\alpha - x_\beta|) \; - \; \mu \; = \; \gamma(|x_\alpha - x_0|) \quad \alpha = 1..n \\ \sum_{\alpha} w_\alpha = 1 \end{cases} \tag{2}$$

It is then possible to compute $\hat{Z}(x_0)$ according to (2) and the local estimation error variance [1]:

$$\sigma_e^2(x_0) \; = \; -\mu \; - \; \gamma(0) \; + \; 2 \sum_{\alpha=1}^{n} w_\alpha \, \gamma(|x_\alpha - x_0|) \tag{3}$$

The kriging estimator $\hat{Z}(x_0)$ is the best linear unbiased estimator of $Z(x_0)$ deduced from the known values. The estimation variance σ_e^2 gives information about the local accuracy of the estimation and for each computed value $\hat{Z}(x_0)$ it is possible to define a confidence interval $[\hat{Z}(x_0) - n_\sigma \sigma_e(x_0), \hat{Z}(x_0) + n_\sigma \sigma_e(x_0)]$ inside which the regionalized variable is present with a given certainty. At the known values the confidence interval collapses (kriging is an exact interpolator), and between those values, its shape depends on the variogram expression i.e. on the regionalized variable spatial structure. It is worth noting that, in general, to apply kriging with success, it is necessary to calibrate the confidence interval, i.e. to determine the link between n_σ and the probability for the regionalized variable to lie inside the confidence interval [3].

3 Experimental Validation

To validate the proposed method, kriging has been applied first to infer the local behaviour of experimental data. The measurements have been carried out in indoor environment at the University of Brussels (ULB). The working frequency was 2 GHz and the transmitting and receiving antennas were vertical $\lambda/2$ electric dipoles. The exact experimental setup and environment are not necessary for the understanding of this study, so that, for sake of conciseness, they will not be described here.

3.1 Global field behaviour

Fig.2 represents the kriging result obtained with the data obtained at the ULB. On this figure, each inferred value $\hat{Z}(x_0)$ has been computed by considering only the two nearest known values in (2) ($n = 2$) and to define the confidence interval $n_\sigma = 1$ has been chosen. It is possible to see that the inferred values $\hat{Z}(x_0)$ are not meaningful by themselves in this case because the initial data were too far away from each others (4λ) to allow fast variations

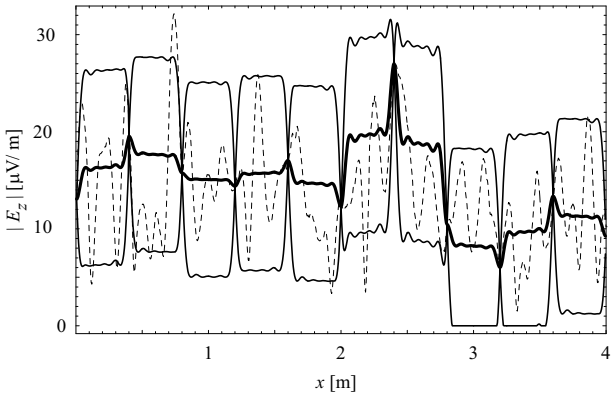

Fig.2. Kriging estimation (thick solid line) and kriging confidence interval (thin solid line) applied to experimental values (dashed line)

prediction. But thanks to the confidence interval, kriging defines variation bounds inside which 92% of the true values lie. Using kriging in combination with a rough sampling scheme allows to predict field envelope variations.

3.2 Local field behaviour

The confidence interval can also be used at the local scale to predict field variations given a set of experimental or numerical data. Let us consider the experimental values drawn on Fig.3 in dashed line obtained on a four wavelengths distance and let us suppose that the only available data are separated $\lambda/3$ (the dashed line is thus supposed unknown). By using classical interpolation through these few values, it is impossible to predict local behaviour, like peak $|E_z|$ values in this interval, because there is no knowledge of the field variation between the measurement points. On the other hand, kriging the data gives between each pair of experimental values a confidence interval for $|E_z|$ with a given certainty (90% in this case). Fig.3 shows the kriging result (computed by considering only the two nearest neighbors in (2) and $n_\sigma = 1.5$) compared to the actual $|E_z|$ values. At the known points, the confidence interval collapses. Between those points, the confidence interval shape on Fig.3 depends on the theoretical variogram shape, and it reflects thus the regionalized variable expected variation as Fig.3 clearly shows. Whatever the sampling rate, kriging allows to take into account the spatial structure of the data to infer point characteristics with a better efficiency.

4 Conclusion

In order to decrease the time necessary to analyze wave propagation in open or closed systems, the spatial statis-

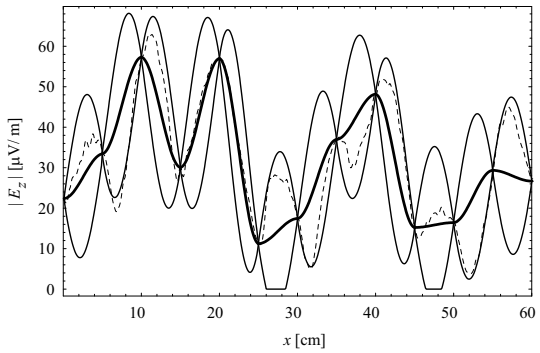

Fig.3 Kriging estimation (thick solid line), kriging confidence interval (thin solid line) and experimental values (dashed line)

tics formalism has been adapted to electromagnetics. In this formalism, the fields are considered as realizations of random functions. It has been shown that their spatial structure analysis can be performed by variographic analysis. Knowing a theoretical expression for the variogram, an interpolation tool called kriging can then be applied to infer fields unknown values. Kriging defines the best linear unbiased estimator of the unknown values and takes into account the spatial structure of the data to predict a confidence interval for each estimation. Finally, kriging has been validated on experimental data obtained in indoor environment.

References

[1] J-P Chilès, P. Delfiner, "Geostatistics, modeling spatial uncertainty", Wiley, New York, 1999

[2] Ph. De Doncker, "Covariance functions for three-dimensional Rayleigh channels", *to be published*

[3] Ph. De Doncker, JM Dricot, R. Meys, M. Hélier, W. Tabbara, "Application of Spatial Statistics to Electromagnetic Wave Propagation", *to be published*