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# Electromagnetic Fields Estimation Using Spatial Statistics

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*The spatial statistics formalism is applied to electromagnetic fields analysis. Fields are considered as realizations of a random function. Their spatial structure is studied by a method known as variographic analysis. To infer unknown values of the fields, an interpolation method called kriging is then used. It is shown how kriging can be performed on experimental or numerical data to speed up the fields estimation process.*

**Keywords** propagation, statistics

## Introduction

Due to the increasing complexity of electromagnetic systems, a strong trend exists to shift from deterministic analysis to statistical approaches. These approaches are, however, always considered as *global* system analysis tools allowing one to predict global characteristics, like fading rates in wireless communications. We propose here to show that statistical tools also exist aiming to obtain *local* information about fields. Basically, the problem that will be tackled can be expressed as follows: Given a set of field values obtained by simulation or experiment, what can be deduced about the *local* behavior of these fields?

Classically, a first answer to this problem would be to obtain field values either by simulations, most often ray-tracing, or by undertaking measurement campaigns. Then, the usual interpolation methods would be used in order to derive the field behavior throughout the area under study. When this area is large compared to the signal wavelength, or

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when it has a complicated geometry, this method is difficult to use. Due to the highly oscillating behavior of fields, several computations or measurements per wavelength would be required for accurate interpolation. In a ray-tracing algorithm, for instance, computation would then drastically increase. In the same way, obtaining extended experimental results requires important work, too time-consuming in practice. Both approaches are thus limited by the dense sampling grid necessary to interpolate fast-varying fields, even when general characteristics of the fields are only required, as is often the case. To speed up electromagnetic fields estimation, a solution would be to deduce those characteristics from only few samples, easy and fast to compute or to measure.

A second classical answer to this problem would be to deduce the field characteristics by postulating their statistical distribution. Log-normal, Rayleigh, or Rice distributions, for instance, are well known in cellular network simulations (Saunders, 1999; Lee, 1999). This approach only provides *global* characteristics like coverage probability or fade duration, but unfortunately no *local* behavior knowledge.

Although local properties of fields are generally not significant from the communication engineering point of view, they can be of the utmost importance in electromagnetic compatibility studies, for instance to determine "hot spots" or field patterns in confined systems like reverberation chambers or electronic device cabinets. The rapid development of MIMO communication systems also fosters the need for local field behavior studies in order to simulate and to design antenna arrays. A lot of work has been carried out in this area during the last few years showing that the local spatial correlation of the fields can be deduced from the power arrival spectrum of the incoming waves and from the antenna-equivalent lengths (De Doncker & Meys, 2004). The importance of the spatial structure of the fields is thus nowadays well recognized, but the existing literature is still restricted to correlation studies, and it does not address the important point of the actual field spatial structure prediction.

The method we propose to infer this local behavior is to use spatial statistics and, more precisely, the methods known as variographic analysis and kriging. As shown later, kriging will provide both an interpolation method and a variation-bound estimator. Of course, due to the highly oscillating behavior of fields, it would be impossible to reproduce their exact fast variations, by any interpolation process. Kriging will not outperform classical methods from the interpolation point of view. The aim of this work is hence neither to optimize the interpolation process nor to define an optimal sampling grid. Spatial statistics inference will rather provide field envelope estimations and, more important, their estimation variance, i.e., information about their confidence interval. Besides having an important impact on the practical inference of fields, it will be shown that spatial statistics can provide a theoretical framework to describe fields, which is currently lacking in electromagnetics.

Kriging has already been applied with success in various circumstances. Before being applied to biology or image processing, spatial statistics were historically developed in the framework of geostatistics (Chils & Delfiner, 1999; Stein, 1999; Wackernagel, 1995) to provide mineral grade estimations for mine exploitation. In this specific case, the mineral grade distribution was considered as a random function (RF) of position, and, thanks to a few mineral samples and a structural tool known as the variogram, the estimated value of this RF was inferred at various locations throughout the mine. The basic idea of the method we propose is to apply this technique to electromagnetic fields: The electric or magnetic field values within a given area may be considered as one realization of a RF. Knowing samples of this RF by means of ray-tracing or measurements, it could be possible by using spatial statistics to infer the RF values over the whole area. Kriging has

already been considered in electromagnetics (Lefebvre et al., 1996), in transmission lines study (Rannou et al., 2001, 2002), for instance, but the whole spatial statistics framework has never been adapted to wave propagation.

In the first section of this paper, it will be shown how the spatial statistics theory can be applied to electromagnetics. Next, variographic analysis and kriging will be introduced and applied to the small scale ("fast") fading case. The new approach will then be applied to experimental results. The measurements have been carried out in an indoor environment, in an NLOS confined situation, at the University of Brussels (ULB). The working frequency was 2 GHz and the transmitting and receiving antennas were vertical $\lambda/2$ electric dipoles. The exact experimental setup and environment are not necessary for the understanding of this study, so, for the sake of conciseness, they will not be described here. In the remainder of the text, a time dependence $e^{j\omega t}$ is supposed and omitted.

## Spatial Statistics

Let us consider the electric field vertical component strength $|E_z|$ drawn in Figure 1 as a function of position along the $x$-axis in the horizontal plane. These values were obtained using a regular experimental sampling, and they can be defined as a collection of values $z(x_\alpha) = |E_z(x_\alpha)|$ at the measurement points $x_\alpha$ ($\alpha = 1 \ldots n$) located in a given region $\mathcal{D}$ (in general, $x$ can be defined as any multidimensional coordinate). The $z(x_\alpha)$ values represent samples of a phenomenon (the electric field) existing throughout the whole region. According to spatial statistics, $\{z(x_\alpha), \alpha = 1 \ldots n\}$ is defined as a set of values of a *regionalized variable* $z(x)$ defined over $\mathcal{D}$.

It is well known that electromagnetic field patterns in propagation problems are very irregular. It seems obvious that the regionalized variable $z(x)$ drawn in Figure 1 cannot be described by any closed form expression. A probabilistic approach must rather be chosen. On the other hand, $z(x)$ also exhibits a spatial structure: The $z(x_\alpha)$ are correlated due to the underlying physical process producing $z(x)$, for instance, small scale fading in Figure 1. To explicitly take into account these two properties (stochastic nature and spatial correlation), each $z(x_\alpha)$ can be considered as the outcome of some underlying

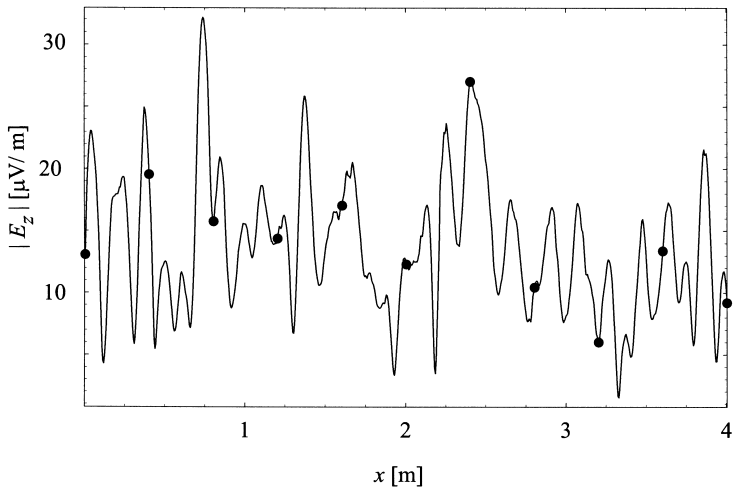


**Figure 1.** Experimental $|E_z|$ values, and data set (dots) considered for kriging in Figure 6.

random process: Each $z(x_\alpha)$ is one realization of a random variable $Z(x_\alpha)$ having some statistical properties. In the same way, the regionalized variable $z(x)$ drawn in Figure 1 becomes one realization of the *random function* $Z(x)$, defined as the infinite collection of random variables $\{Z(x_\alpha), x_\alpha \in \mathcal{D}\}$.[1]

From a statistical point of view, the RF $Z(x)$ is meaningful to determine the statistical properties of the process, but only a particular realization $z(x)$ is accessible. To perform inference on the process using one single realization, it is necessary to make the ergodicity assumption as usual in propagation problems (Lee, 1993), i.e., to assume that the single spatial realization $z(x)$ is representative of the statistical properties of each random variable $Z(x_\alpha)$. Moreover, to apply spatial statistics tools like kriging, the mean and the covariance of the RF $Z(x)$ have to be stationary over $\mathcal{D}$ to be meaningful i.e.,

$$\begin{aligned} &E\{Z(x)\} = m, \\ &E\{[Z(x) - m][Z(x+h) - m]\} = C(h), \end{aligned} \tag{1}$$

where $E\{\}$ denotes the mathematical expectation and $C(h)$ the covariance function. Equation (1) states that the mean and the covariance of $Z(x)$ are to be independent of position in $\mathcal{D}$. In this case, the RF is said to be *second-order stationary*. We shall restrict ourselves here to this class of RF. It is obvious that in propagation problems the mean electric field strength, for instance, is not always stationary when in $\mathcal{D}$ the distance to the transmitter greatly varies. In this case, either the basic spatial statistics tools are to be applied on subsets of $\mathcal{D}$, or generalized tools must be considered (Chils & Delfiner, 1999; Stein, 1999; Wackernagel, 1995).

## Variographic Analysis

### *Theoretical Background*

The spatial structure of the regionalized variable $z(x)$ can be studied by looking at the associated RF $Z(x)$ moments. By restricting ourselves to second-order stationary RF we have chosen to focus only on the first two moments of $Z(x)$. Its first moment (the mathematical expectation) is under hypothesis (1) a constant and it does not give any information about the spatial structure of the regionalized variable. On the other hand, the second moment allows us to study the similarity or dissimilarity between pairs of values as a function of their spatial lag. In the spatial statistics framework, the second moment study is called the variographic analysis.

The variogram[2] $\gamma(h)$ measures the dissimilarity between values of $Z(x)$ at $x$ and $x + h$:

$$\gamma(h) = \frac{1}{2}\mathrm{Var}\{Z(x+h) - Z(x)\}, \tag{2}$$

where Var{} indicates the variance of a random variable. Using (1), $E\{Z(x+h)-Z(x)\} = 0$ and the variogram can be written in a more intuitive form:

$$\gamma(h) = \frac{1}{2}E\{[Z(x+h) - Z(x)]^2\}. \tag{3}$$

[1]In the remainder of the text, capital letters will denote random functions and small letters their realizations.

[2]Using definition (2), $\gamma(h)$ is sometimes called the *semi*-variogram.

It is important to note that, in general, $x$ and $h$ are vectors, so that $\gamma(h)$ depends on both distance and direction. Since the variogram reflects the correlation properties of $Z(x)$, it is a powerful tool to study the spatial structure of experimental or numerical data. It allows us to detect inhomogeneities, anomalies, or eventual anisotropy in the multidimensional case (Wackernagel, 1995). The general behavior of $\gamma(h)$ is drawn in Figure 2 in the unidimensional case. Since, according to (3), the variogram represents the average square increment between pairs of values spaced by the lag $h$, it generally increases with distance. The variogram behavior near the origin indicates the data regularity. If $\gamma(0) \neq 0$, the variogram is said to present a *nugget effect* reflecting either a physical phenomenon at smaller scale which has not been well resolved by the experiment or computation (neighboring values have large dissimilarities, indicating that the sampling rate was not sufficient), or an experimental or computational error too important compared to the regionalized variable scale (in this case it can be shown that, under mild assumptions, $\gamma(0)$ is the experimental error variance). At a distance called the *range*, the variogram reaches a *sill* indicating the absence of spatial structure, i.e., the absence of correlation. The sill level is a measure of the variation amplitude of the data. A variogram can exhibit several sills showing that several phenomena acting at different scales are underlying the data. In the multidimensional case, a difference in nugget effect, range, or sill with respect to the direction allows the detection of anisotropy or preferential directions.

It is worth noting that under the second-order stationarity assumption

$$\gamma(h) = C(0) - C(h) \tag{4}$$

and the variogram is equivalent to the covariance. However, the variogram analysis is preferable on one hand because it can be generalized to nonstationary RF and on the other hand because it does not require the expectation value knowledge which must be estimated from the data for the covariance, inducing a possible bias (Chils & Delfiner, 1999).

The variogram defined by (2) relies on the knowledge of the whole random process $Z(x)$. However, in practice, only one realization of $Z(x)$ is accessible. It is necessary to

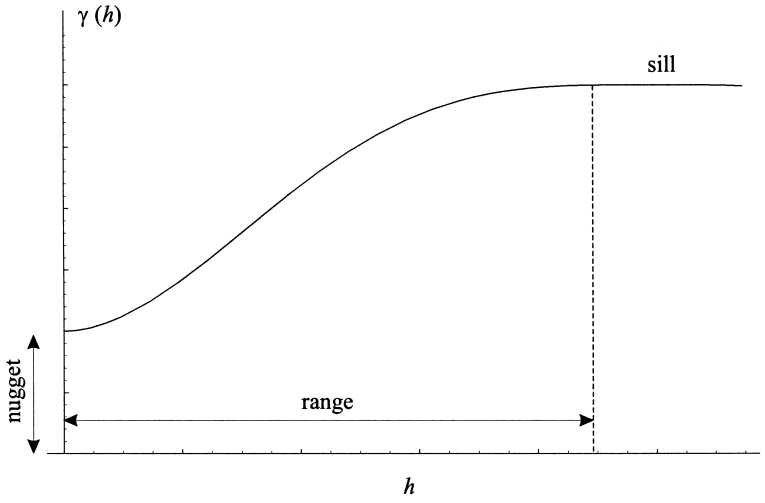


**Figure 2.** Typical variogram showing the nugget effect, the range, and the sill.

estimate $\gamma(h)$ from this realization. An unbiased estimation of the variogram is given by the experimental variogram

$$\hat{\gamma}(h) = \frac{1}{2N_h} \sum_{|x_\alpha - x_\beta| = h} [z(x_\alpha) - z(x_\beta)]^2, \tag{5}$$

where the sum is performed over all the experimental points spaced by the lag $h$, and where $N_h$ is the count of pairs spaced by this lag.

### *Application to Wave Propagation*

To each physical process underlying the RF corresponds an experimental variogram, and $\hat{\gamma}(h)$ can be viewed as the common signature of regionalized variables sharing the same origin. In wave propagation, different experimental variograms will be found for different kinds of propagation: log-normal shadowing, Rayleigh, or Rice small scale fading, for instance. It will also depend on the power arrival spectrum of the waves and on the receiving antenna equivalent length, as found in the usual correlation studies (De Doncker & Meys, 2004). The experimental variogram corresponding to Figure 1 is drawn in Figure 3 with the dotted line. It does not exhibit any nugget effect and its range is about $\lambda/2$ (where $\lambda$ is the free-space wavelength), as expected from the classical correlation studies. Its oscillating behavior at the sill reflects the oscillating nature of the data in Figure 1.

Since the variogram is a signature of the propagation mechanism, it is possible on physical grounds to theoretically derive its mathematical expression (eventually via the RF covariance). In the case of confined small-scale fading in three-dimensional Rayleigh channels, and using vertical dipole antennas, it is possible to show using (4) and De Doncker (2003) that the variogram for $|E_z|$ in the horizontal plane is given by

$$\gamma(h) = \Gamma\left(1 - \frac{2}{\pi}\rho(h)\arcsin(\rho(h)) - \sqrt{1-\rho(h)}\right), \tag{6}$$

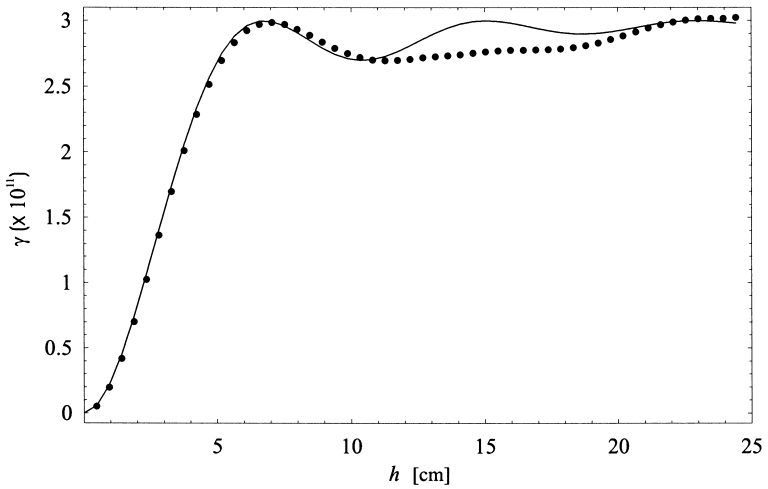


**Figure 3.** Theoretical (solid line) and experimental (dotted line) variograms corresponding to the data of Figure 1.

where $\Gamma$ is a scale factor to be adjusted and depending on the mean $|E_z|$ value inside $\mathcal{D}$, and where $\rho(h)$ is the correlation function defined with a good accuracy by (De Doncker, 2003; De Doncker & Meys, 2004)

$$\rho(h) = \frac{3}{2}\frac{\sin kh}{kh}\left(1 - \frac{1}{(kh)^2}\right) + \frac{3}{2}\frac{\cos kh}{(kh)^2}, \tag{7}$$

where $k$ is the free-space wave number.

This theoretical variogram is also drawn in Figure 3. The agreement between the theoretical and experimental variograms is perfect for low $h$. For $h$ values beyond the range, the discrepancy is more important due to the uncertainty in $\hat{\gamma}(h)$. It can be shown (Chils & Delfiner, 1999) that the relative error on one value $\hat{\gamma}(h)$ is given by $2/\sqrt{N_h}$, where $N_h$ is the count of pairs used to compute $\hat{\gamma}(h)$. For high $h$, the number of available experimental pairs decreases and the accuracy drops.

The variogram is not just a diagnostic tool useful to explore experimental or numerical data; it also plays a major role in unknown values inference using kriging. As a matter of fact, the choice of a theoretical variogram is a very important task for kriging. To each propagation channel (power arrival spectrum, antennas) corresponds a variogram which can be, in principle, derived on physical grounds. Using the correlation results in De Doncker and Meys (2004), for instance, it is possible to build a set of variograms related to different propagation scenarios. Conversely, fitting a theoretical variogram on experimental data can stress what the main propagation mechanisms are in a measurement campaign. The good agreement in Figure 3 shows, for instance, that the power arrival spectrum was almost isotropic in this case. To find a priori the right variogram could be a difficult task, however. At the beginning of any field analysis, a kind of variogram calibration is then mandatory.

## Kriging

Let us consider a set of field values obtained numerically or experimentally. Inferring field values from these data is a classical interpolation problem which can be addressed using either exact interpolators, like lagrangian polynomials, or least-square methods minimizing the interpolation error *at* the known values. Both methods have two important drawbacks: First, they do not take into account the spatial structure of the regionalized variable being interpolated, and, more important, they do not give any information about the local interpolation accuracy. Let us consider, for instance, the dots drawn in Figure 1 obtained by choosing $|E_z|$ values spaced by 60 cm ($4\lambda$). It is obvious that those points only give very little information about the whole regionalized variable. Any classical interpolation defined through these points would not give much more information about the small scale variations. It would be necessary to use a much denser sampling scheme to obtain accurate results.

To be able to deduce useful information from the few data drawn in Figure 1, it is necessary to know the regionalized variable local spatial structure. As shown in the previous section, this structure can be given by the variogram $\gamma(h)$. The process of interpolation taking into account spatial structure using the variogram is called *kriging*.

The kriging theory is defined in the probabilistic framework of regionalized variables. Let $\hat{Z}(x_0)$ be an estimation of the unknown variable $Z(x_0)$ at a given location $x_0$ in $\mathcal{D}$. For this estimation to be linear in the known values at the experimental or computational

points $x_\alpha$, it is written as a sum of $n$ variables $Z(x_\alpha)$ in the $x_0$ neighborhood (the so-called *kriging neighborhood*):

$$\hat{Z}(x_0) = \sum_{\alpha=1}^{n} w_\alpha Z(x_\alpha), \tag{8}$$

where $w_\alpha$ are coefficients to be determined. Generally, to decrease the computation time, the kriging neighborhood is chosen in the close vicinity of $x_0$ ($\hat{Z}(x_0)$ is written in terms of variables $Z(x_\alpha)$ close to $x_0$), and $n$ is usually very low.

Next, imposing this estimator to be unbiased,

$$E\{\hat{Z}(x_0) - Z(x_0)\} = 0, \tag{9}$$

and assuming the second-order stationarity, the condition $\sum_\alpha w_\alpha = 1$ must be satisfied. Finally, this estimator is chosen to be optimal in the sense that it minimizes the estimation error variance at $x_0$:

$$\sigma_e^2(x_0) = \mathrm{Var}\{\hat{Z}(x_0) - Z(x_0)\}. \tag{10}$$

By minimizing $\sigma_e^2(x_0)$ with the constraint $\sum_\alpha w_\alpha = 1$ introduced by a Lagrange multiplier $\mu$, a kriging system for the $w_\alpha$ and $\mu$ coefficients is obtained (Chils & Delfiner, 1999; Stein, 1999; Wackernagel, 1995):

$$\begin{cases} \displaystyle\sum_{\beta=1}^{n} w_\beta \gamma(|x_\alpha - x_\beta|) - \mu = \gamma(|x_\alpha - x_0|), \quad \alpha = 1 \ldots n, \\ \displaystyle\sum_{\alpha} w_\alpha = 1, \end{cases} \tag{11}$$

where $\gamma$ is the theoretical variogram defined in the previous section. To solve this system, $\gamma$ has to be chosen. For the dots in Figure 1, for instance, as stated above, it is possible to argue that the associated theoretical variogram is given by (6). To adjust $\Gamma$ in (6), it is sufficient to compute one $\hat{\gamma}(h)$ value according to (5), using the few dots in Figure 1.

It is then possible to compute $\hat{z}(x_0)$ for the particular realization considered according to

$$\hat{z}(x_0) = \sum_{\alpha=1}^{n} w_\alpha z(x_\alpha) \tag{12}$$

and the local estimation error variance (Chils & Delfiner, 1999; Stein, 1999; Wackernagel, 1995):

$$\sigma_e^2(x_0) = -\mu - \gamma(0) + 2\sum_{\alpha=1}^{n} w_\alpha \gamma(|x_\alpha - x_0|). \tag{13}$$

The estimation variance $\sigma_e^2(x_0)$ gives information about the local accuracy of the estimation. For each computed value $\hat{z}(x_0)$ it is possible to define a confidence interval $[\hat{z}(x_0) - n_\sigma \sigma_e(x_0), \hat{z}(x_0) + n_\sigma \sigma_e(x_0)]$ where the regionalized variable is present with a given certainty.

The kriging estimator $\hat{z}(x_0)$ is the best linear unbiased estimator of $z(x_0)$ deduced from the known values. It has been obtained by minimizing the *local* error variance and it takes into account the spatial structure of the regionalized variable by means of the variogram. If the known values are close to each other, they are correlated, and $\hat{z}(x_0)$ will closely follow the true regionalized variable. Otherwise, due to the lack of information, $\hat{z}(x_0)$ will only give a rough estimation of the true values, and the estimation variance is of the utmost importance to predict the regionalized variable variation bounds. A typical kriging result between two known values is drawn in Figure 4. At each location $x_0$ between those points a pair $\{\hat{z}(x_0), \sigma_e^2(x_0)\}$ is computed using (11)–(13). As shown in Figure 4, at the known values the confidence interval collapses (kriging is an exact interpolator), and between those values, its shape depends on the variogram expression, i.e., on the regionalized variable spatial structure.

In general, to apply kriging with success, it is necessary to calibrate the confidence interval, i.e., to determine the link between $n_\sigma$ and the probability for the regionalized variable to lie inside the confidence interval. It is possible to show that this link is given by (Chils & Delfiner, 1999)

$$\begin{cases} n_\sigma = \sqrt{\dfrac{4}{9(1-p)}}, & n_\sigma > \sqrt{8/3}, \\ n_\sigma = \sqrt{\dfrac{4}{4-3p}}, & n_\sigma < \sqrt{8/3}, \end{cases} \tag{14}$$

where $p$ is the desired probability that the regionalized variable lies in the confidence interval. It has been found on practical implementations that due to the very general assumptions made in its derivation, this expression is very conservative in the small scale fading case. To obtain a more accurate estimation it is preferable to calibrate the confidence interval for each class of propagation study. Figure 5 shows this calibration carried out on the data of Figure 1 compared to the theoretical expression (14). The experimental points on this figure have been obtained by computing the proportion of experimental $|E_z|$ values lying inside the confidence interval as a function of $n_\sigma$. Very similar results have been obtained with other data, and it is possible to argue that in the Rayleigh small scale fading case, using $n_\sigma = 1.5$, the probability for the regionalized variable to lie inside the confidence interval is higher than 90%. For each propagation

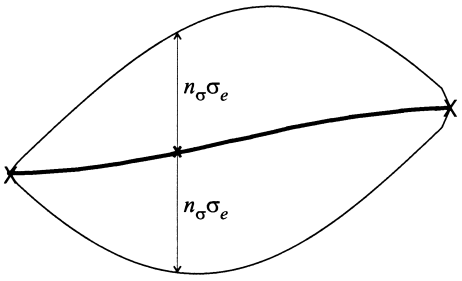


**Figure 4.** Typical kriging result between two known values (crosses). Kriging estimator $\hat{z}(x)$ (thick line) and confidence interval $[\hat{z}(x) - n_\sigma\sigma_e(x), \hat{z}(x) + n_\sigma\sigma_e(x)]$ (solid lines).

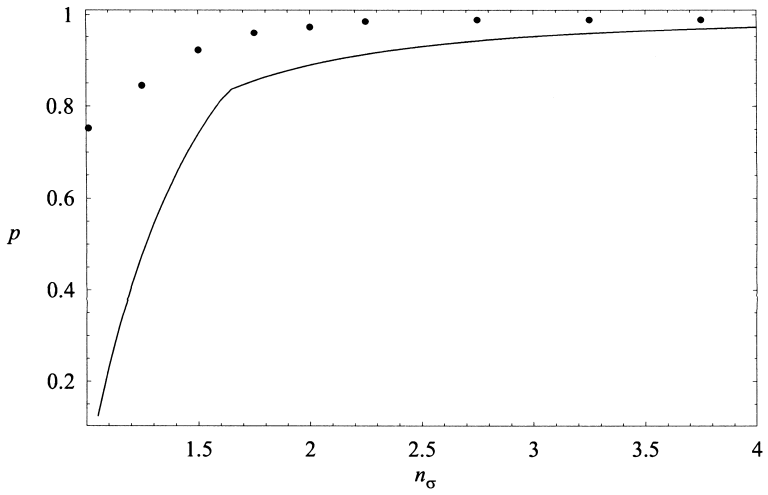


**Figure 5.** Experimental (dots) and theoretical (solid line) kriging certainty level calibration.

mechanism and for each desired certainty level, either (14) (or another eventual theoretical expression) can be used, or a calibration must be carried out before applying kriging.

## Applications

To validate the proposed method and to show its utility, kriging has been applied to infer the local behavior of experimental data.

### *The Kriging Algorithm*

Given a set of experimental or numerical field values, kriging is applied in three steps.

1. A theoretical expression for the variogram is chosen according to the propagation mechanism under study.
2. Using the data, point values of the experimental variogram are computed according to (5), and the theoretical variogram is adjusted to fit these values.
3. At each location $x_0$ where the field is to be inferred, (11)–(13) are applied to find the kriging estimation and the corresponding variance. Since $n$ in (11) is usually chosen very low, this last step can be most often carried out analytically.

### *Sparse Data Case*

Figure 6 shows the kriging result obtained with the dots in Figure 1. In this figure, each inferred value $\hat{z}(x_0)$ has been computed by considering only the two nearest known values in (8) ($n = 2$). To define the confidence interval $n_\sigma = 1$ has been chosen. It is possible to see that the inferred values $\hat{z}(x_0)$ are not meaningful by themselves in this case because the initial data were too far away from each other ($4\lambda$) to allow small scale variation prediction. But, thanks to the confidence interval, kriging defines variation bounds wherein 92% of the true values lie. Using kriging in combination with a rough sampling scheme thus allows us to predict field envelope variations.

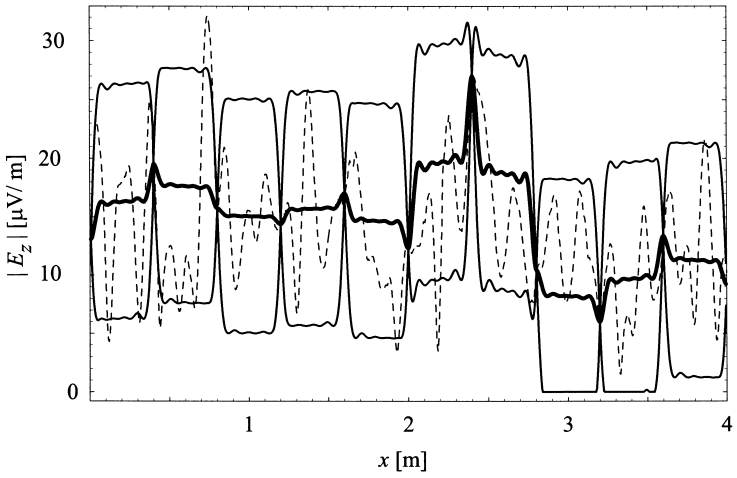


**Figure 6.** Kriging applied to the data set of Figure 1. Kriging estimator (thick line), confidence interval (solid lines), and experimental values (dashed line).

### *Dense Data Case*

As shown in Figure 6, kriging gives large-scale prediction of the field envelope when only few data are known. The confidence interval can also be used at the local scale to predict variations of the fields given a denser set of experimental or numerical data. Let us consider the experimental values drawn in Figure 7 (dashed line) obtained on a four-wavelength distance, and let us suppose that the only available data are spaced by $\lambda/3$ (the dashed line is thus supposed unknown). By using classical interpolation through these few values, it is impossible to predict any local behavior, like peak $|E_z|$ values in this interval, because there is no knowledge of the field variations between

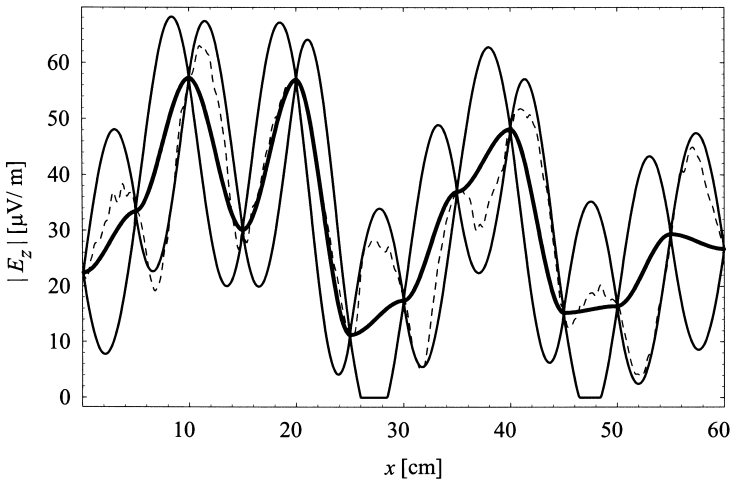


**Figure 7.** Kriging estimator (thick line), confidence interval (solid lines), and experimental values (dashed line).

the measurement points. On the other hand, kriging the data gives between each pair of experimental values a confidence interval for $|E_z|$ with a given certainty (90% in this case). Figure 7 shows the kriging result (computed by considering only the two nearest neighbors in (8) and $n_\sigma = 1.5$) compared to the actual $|E_z|$ values. At the known points, the confidence interval collapses. Between those points, the confidence interval shape in Figure 7 depends on the theoretical variogram shape (6). It thus reflects the expected shape of the regionalized variable, as Figure 7 clearly shows.

## Conclusion

The spatial statistics formalism has been adapted to electromagnetic wave propagation. The fields are considered as regionalized variables corresponding to one realization of a random function. The spatial structure analysis of these regionalized variables has been performed by variographic analysis. Knowing a closed form expression for the variogram, an interpolation tool called kriging has then been applied to infer unknown values of the regionalized variable. Kriging defines the best linear unbiased estimator of the unknown values and, more important, it takes into account the spatial structure of the data to predict a confidence interval for each estimation.

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