A CONCISE OVERVIEW OF APPLICATIONS OF ANT COLONY OPTIMIZATION

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Ant colony optimization (ACO) [1–3] is a metaheuristic for solving hard combinatorial optimization problems inspired by the indirect communication of real ants. In ACO algorithms, (artificial) ants construct candidate solutions to the problem being tackled, making decisions that are stochastically biased by numerical information based on (artificial) pheromone trails and available heuristic information. The pheromone trails are updated during algorithm execution to bias the ants search toward promising decisions previously found. The article titled ***Ant Colony Optimization*** gives a detailed overview of the main concepts of ACO.

Despite being one of the youngest metaheuristics, the number of applications of ACO algorithms is very large. In principle, ACO can be applied to any combinatorial optimization problem for which some iterative solution construction mechanism can be conceived. Most applications of ACO deal with $\mathcal{NP}$-hard combinatorial optimization problems, that is, with problems for which no polynomial time algorithms are known. ACO algorithms have also been extended to handle problems with multiple objectives, stochastic data, and dynamically changing problem information. There are extensions of the ACO metaheuristic for dealing with problems with continuous decision variables, as well.

This article provides a concise overview of several noteworthy applications of ACO algorithms. This overview is necessarily incomplete because the number of currently available ACO applications goes into the hundreds. Our description of the applications follows the classification used in the 2004 book on ACO by Dorigo and Stützle [3] but extending the list there with many recent examples. Tables 1 and 2 summarize these applications.

APPLICATIONS TO $\mathcal{NP}$-HARD PROBLEMS

ACO was primarily intended for solving combinatorial optimization problems, among which $\mathcal{NP}$-hard problems are the most challenging ones. In fact, no polynomial-time algorithms are known for such problems, and therefore heuristic techniques such as ACO are often used for generating high-quality solutions in reasonable computation times.

Routing Problems

Routing problems involve one or more agents visiting a predefined set of locations, and the objective function and constraints depend on the order in which the locations are visited. Perhaps the best-known example is the traveling salesman problem (TSP) [104,105]. In fact, the first ACO algorithm, ant system (AS) [4,5,106,107], was first tested using this problem. Although AS could not compete with state-of-the-art algorithms for the TSP, it was the starting point for the development of various high performing ACO algorithms. The application of AS to the TSP also stimulated the application of ACO to other routing and combinatorial problems.

For instance, ACO has obtained very good results for the sequential ordering problem, an extension of asymmetric TSP with precedence constraints among nodes. At the time it was proposed by Gambardella and Dorigo [18], the algorithm was the best available algorithm for this problem, improving upon many best-known solutions. Recently, stochastic sampling has been integrated into a Beam-ACO algorithm for the TSP with time windows [19], which is an extension of the classical TSP with time window constraints; Beam-ACO is a combination of ACO algorithms with beam-search [32].

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Table 1. Applications of ACO Algorithms to $\mathcal{NP}$-hard Problems

Problem Type	Problem Name	References
Routing	Traveling salesman	Dorigo *et al*. [4,5]
		Dorigo and Gambardella [6]
		Stützle and Hoos [7,8]
	Vehicle routing (VRP)	Bullnheimer *et al*. [9]
		Reimann *et al*. [10]
		Rizzoli *et al*. [11]
	VRP with time windows	Gambardella *et al*. [12]
	VRPMTWMV	Favoretto *et al*. [13]
	VRP with loading constraints	Doerner *et al*. [14]
		Fuellerer *et al*. [15,16]
	Team orienteering	Ke *et al*. [17]
	Sequential ordering	Gambardella and Dorigo [18]
	TSP with time windows	López-Ibáñez and Blum [19]
Scheduling	Single machine	Den Besten *et al*. [20]
		Merkle and Middendorf [21,22]
		Meyer and Ernst [23]
		Liao and Juan [24]
		Meyer [25]
	Flow shop	Stützle [26]
		Rajendran and Ziegler [27]
	Industrial scheduling	Gravel *et al*. [28]
	Project scheduling	Merkle *et al*. [29]
	Group shop	Blum [30]
	Job shop	Blum [30]
		Huang and Liao [31]
	Open shop	Blum [32]
	Car sequencing	Khichane *et al*. [33]
		Solnon [34]
		Morin *et al*. [35]
Subset	Multiple knapsack	Leguizamón and Michalewicz [36]
		Ke *et al*. [37]
	Maximum independent set	Leguizamón and Michalewicz [36]
	Redundancy allocation	Liang and Smith [38]
	Weight constraint graph tree partitioning	Cordone and Maffioli [39]
	Bin packing	Levine and Ducatelle [40]
	Set covering	Lessing *et al*. [41]
	Set packing	Gandibleux *et al*. [42]
	l-cardinality trees	Blum and Blesa [43]
	Capacitated minimum spanning tree	Reimann and Laumanns [44]
	Maximum clique	Solnon and Fenet [45]
	Multilevel lot-sizing	Pitakaso *et al*. [46,47]
		Almeder [48]
	Edge-disjoint paths	Blesa and Blum [49]
	Feature selection	Sivagaminathan and Ramakrishnan [50]
	Multicasting ad-hoc networks	Hernández and Blum [51]
Assignment and layout	Quadratic assignment	Maniezzo *et al*. [52,53]
		Stützle and Hoos [8]
	Graph coloring	Costa and Hertz [54]
	Generalized assignment	Lourenço and Serra [55]
	Frequency assignment	Maniezzo and Carbonaro [56]

Table 1. (*Continued*)

Problem Type	Problem Name	References
	Constraint satisfaction	Solnon [57,58]
	Course timetabling	Socha *et al*. [59,60]
	Ambulance location	Doerner *et al*. [61]
	MAX-SAT	Pinto *et al*. [62]
	Assembly line balancing	Bautista and Pereora [63]
	Simple assembly line balancing	Blum [64]
	Supply chain management	Silva *et al*. [65]
Machine learning	Bayesian networks	De Campos *et al*. [66,67]
		Pinto *et al*. [68]
	Classification rules	Parpinelli *et al*. [69]
		Martens *et al*. [70]
		Otero *et al*. [71]
Bioinformatics	Shortest common supersequence	Michel and Middendorf [72,73]
	Protein folding	Shmygelska and Hoos [74]
	Docking	Korb *et al*. [75,76]
	Peak selection in biomarker identification	Ressom *et al*. [77]
	DNA sequencing	Blum *et al*. [78]
	Haplotype inference	Benedettini *et al*. [79]

Table 2. Applications of ACO Algorithms to “Nonstandard” Problems

Problem Type	Problem Name	References
Multiobjective	Scheduling	Iredi *et al*. [80]
	Portfolio selection	Doerner *et al*. [81,82]
	Quadratic assignment	López-Ibáñez *et al*. [83,84]
	Knapsack	Alaya *et al*. [85]
	Traveling salesman	García-Martínez *et al*. [86]
	Activity crashing	Doerner *et al*. [87]
	Orienteering	Schilde *et al*. [88]
Continuous	Neural networks	Socha and Blum [89]
	Test problems	Socha and Dorigo [90]
Stochastic	Probabilistic TSP	Bianchi *et al*. [91]
		Bianchi and Gambardella [92]
		Balaprakash *et al*. [93]
	Vehicle routing	Bianchi *et al*. [94]
	Screening policies	Brailsford *et al*. [95]
Dynamic	Network routing	Di Caro and Dorigo [96]
		Di Caro *et al*. [97]
	Dynamic TSP	Guntsch and Middendorf [98,99]
		Eyckelhof and Snoek [100]
		Sammound *et al*. [101]
	Vehicle routing	Montemanni *et al*. [102]
		Donati *et al*. [103]

ACO algorithms have been successful in tackling various variants of the vehicle routing problem (VRP). The first application of ACO to the capacitated VRP (CVRP) was due to Bullnheimer *et al*. [9]. More recently, Reimann *et al*. [10] proposed a particular ACO algorithm (D-Ants) for the capacitated VRP. Gambardella *et al*. [12] introduced

MACS–VRPTW, an ACO algorithm for the VRP with time window (VRPTW) constraints, which reached state-of-the-art results when it was proposed. Favaretto *et al*. [13] proposed an ACS algorithm for a variant of the VRP with multiple time windows and multiple visits (VRPMTWMV). Fullerer *et al*. [15] used an ACO algorithm for a problem that combines the two-dimensional packing and the capacitated vehicle routing problem, showing that it outperforms a tabu search (TS) algorithm. In this problem, items of different sizes and weights are loaded in vehicles with a limited weight capacity and limited two-dimensional loading surface, and then they are distributed to the customers. Other variants of VRP with different loading constraints have also been tackled by means of ACO [14,16].

Ke *et al*. [17] have recently proposed an ACO approach to the team orienteering problem (TOP), where the goal is to find the set of paths from a starting point to an ending point that maximizes the reward obtained by visiting certain locations taking into account that there are restrictions on the length of each path.

Scheduling Problems

Scheduling problems concern the assignment of jobs to one or various machines over time. Input data for these problems are processing times but also often additional setup times, release dates and due dates of jobs, measures for the jobs' importance, and precedence constraints among jobs. Scheduling problems have been an important application area of ACO algorithms, and the currently available ACO applications in scheduling deal with many different job and machine characteristics.

The single-machine total weighted tardiness problem (SMTWTP) has been tackled by both den Besten *et al*. [20] and Merkle and Middendorf [21,22] using variants of ACS (ACS-SMTWTP). In ACS-SMTWTP, a solution is determined by a sequence of jobs. The positions of the sequence are filled in their canonical order, that is, first a job is assigned to position 1, next a job to position 2, and so on, until position n. Pheromone trails are defined as the desirability of scheduling job j at position i, a pheromone trail definition that is used in many ACO applications to scheduling problems [20,26,108,109]. Merkle and Middendorf [21] used sophisticated heuristic information and an algorithmic technique called *pheromone summation rule*, which has proven to be useful in many applications of ACO to scheduling problems. On the other hand, den Besten *et al*. [20] combined ACS-SMTWTP with a powerful local search algorithm, resulting in one of the best algorithms available for this problem in terms of solution quality. Another application of ACO to a variant of this problem with sequence-dependent setup times has recently been studied by Liao and Juan [24]. Meyer and Ernst [23] and Meyer [25] studied the integration of constraint programming techniques into ACO algorithms using a single-machine problem with sequence-dependent setup times, release dates, and deadlines for jobs, as a case study.

ACO algorithms have also been proposed for the permutation flow-shop problem (FSP). The first approach is due to Stützle [26], who proposed a hybrid between $\mathcal{MM}$AS and ACS. Later, Rajendran and Ziegler [27] improved its performance by introducing the pheromone summation rule. For this problem, however, the results of existing ACO algorithms are behind the current state-of-the-art algorithms. This is also the case for the well-known job-shop problem [30], although recent results hybridizing ACO and TS seem promising [31]. Nevertheless, for various other scheduling problems ACO algorithms are among the best performing algorithms available nowadays. Beam-ACO, the hybrid between beam search and ACO, is a state-of-the-art algorithm for open shop scheduling [32]. In addition, a variant of $\mathcal{MM}$AS obtained excellent results in the group shop problem [30].

Another scheduling problem where ACO obtained excellent results is the resource-constrained project scheduling problem, in which a set of activities must be scheduled, subject to resource constraints and precedence constraints among the activities, such that the last activity is completed as early as possible. At the time of its publication, the

ACO algorithm proposed by Merkle *et al*. [29] was the best available.

Finally, state-of-the-art results have been obtained in the car sequencing problem by the ACO algorithm proposed by Solnon [34], and these results have been further improved by Morin *et al*. [35] by means of a specialized pheromone model. The car sequencing problem has also been used as an example application by Khichane *et al*. [33] to explore the integration of constraint programming techniques into ACO algorithms.

Subset Problems

The goal in subset problems is, generally speaking, to find a subset of the available items that minimizes a cost function defined over the items and that satisfies a number of constraints. This is a wide definition that can include other classes of problems. There are, however, two characteristic properties of the solutions to subset problems: The order of the solution components is irrelevant, and the number of components of a solution may differ from solution to solution.

An important subset problem is the set covering problem (SCP). Lessing *et al*. [41] compared the performance of a number of ACO algorithms for the SCP, with and without the usage of a local search algorithm based on 3-flip neighborhoods [110]. The best performance results were obtained, as expected, when including local search. For a large number of instances, the computational results were competitive with state-of-the-art algorithms for the SCP.

Leguizamón and Michalewicz [36] proposed the first ACO applications to the multiple knapsack and to the maximum independent set problems, which were, however, not competitive with the state-of-the-art. Currently, the best performing ACO algorithm for the multiple knapsack problem is due to Ke *et al*. [37]. Levine and Ducatelle [40] adapted $\mathcal{MM}$AS to the well-known bin-packing problem and compared its performance with the hybrid grouping genetic algorithm [111], and with Martello and Toth's reduction method [112]. The $\mathcal{MM}$AS algorithm outperformed both, obtaining better solutions in a much shorter time. Solnon and Fener [45] carried out a comprehensive study for the maximum clique problem. Their conclusion was that ACO combined with appropriate local search can match the quality of state-of-the-art heuristics. Blesa and Blum [49] applied ACO to the problem of finding edge-disjoint paths in networks, and found the performance of the proposed ACO superior in terms of both solution quality and computation time when compared with a multistart greedy algorithm. Another interesting application is the work of Sivagaminathan and Ramakrishnan [50], which discusses how ACO may be hybridized with neural networks for optimizing feature selection in multivariate analysis.

Cordone and Maffioli [39] introduced the weight constrained graph tree partition problem, and tested different variants of ACS with and without local search. Blum and Blesa [43] tackled the edge-weighted k-cardinality tree problem (or k-minimum spanning tree), where the goal is to find a tree over a graph with exactly k edges minimizing the sum of the weights. They compared a $\mathcal{MM}$AS variant, TS, and an evolutionary algorithm. Their results showed that none of the approaches was superior to the others in all instance classes tested, and that $\mathcal{MM}$AS was better suited for instances where the value of k was much smaller than the number of vertices.

A subset problem closely related to the CVRP is the capacitated minimum spanning tree problem, which has been effectively tackled by a hybrid ACO algorithm [44] based on a previous ACO algorithm for the CVRP [10]. More recently, Hernández and Blum [51] considered the minimization of power consumption when multicasting in static wireless ad-hoc networks. This problem can be stated as an $\mathcal{NP}$-hard combinatorial problem, where the goal is to find a directed tree over the network of nodes. Their proposed ACO algorithm outperforms existing algorithms for several variants of this problem.

Finally, a class of problems for which ACO has recently shown competitive results is that of multilevel lot-sizing with [46,48] and without capacity constraints [47]. In these problems, a subset of items is scheduled for production at each time interval, and the goal is to minimize the cost of producing the items, taking into account several constraints and relations between the items.

Assignment and Layout Problems

In assignment problems, a set of items has to be assigned to a given number of resources subject to some constraints. Probably, the most widely studied example is the quadratic assignment problem (QAP), which was among the first problems tackled by ACO algorithms [5,52,53]. Various high-performing ACO algorithms for the QAP have followed this initial work. Among them is the approximate nondeterministic tree search (ANTS) algorithm by Maniezzo [113] a combination of ACO with tree search techniques involving the usage of lower bounds to rate solution components and to prune extensions of partial solutions. The computational results of ANTS on the QAP were very promising. Another high-performing ACO algorithm is the $\mathcal{MAX}$–$\mathcal{MIN}$ ant system ($\mathcal{MM}$AS) proposed by Stützle and Hoos [8], which is among the best algorithms available for large, structured instances of the QAP.

The ANTS algorithm has also been applied to the frequency assignment problem (FAP), in which frequencies have to be assigned to links and there are constraints on the minimum distance between the frequencies assigned to each pair of links. ANTS showed good performance on some classes of FAP instances in comparison with other approaches [56]. Other applications of ACO to assignment problems include university course timetabling [59,60] and graph coloring [54]. The work of Solnon [57,58] applies ACO algorithms to the general class of constraint satisfaction problems (CSPs); in fact, decision variants of problems such as graph coloring and frequency assignment can be seen as cases of CSPs. Within this class, Pinto *et al*. [62] studied the application of ACO to regular and dynamic MAX-SAT problems.

Another notable example is the generalized assignment problem, where a set of tasks have to be assigned to a set of agents with a limited total capacity, minimizing the total assignment cost of tasks to agents. The $\mathcal{MM}$AS algorithm proposed by Lourenço and Serra [55] was, at the time of its publication, close to the state-of-the-art algorithm for this problem. More recently, Doerner *et al*. [61] tackled a real-world problem related to ambulance locations in Austria by means of an ACO algorithm; and Blum [64] has shown that the hybrid between beam search and ACO, Beam-ACO, is a state-of-the-art algorithm for simple assembly line balancing. In the section titled "Industrial Applications," we mention an industrial application of ACO to assembly line balancing. Finally, Silva *et al*. [65] have used ACO for a complex supply chain management problem that combines aspects of the generalized assignment, scheduling, and vehicle routing problems.

Machine Learning Problems

Diverse problems in the field of machine learning have been tackled by means of ACO algorithms. Notable examples are the work of Parpinelli *et al*. [69] and Martens *et al*. [70] on applying ACO to the problem of learning classification rules. This work was later extended by Otero *et al*. [71] in order to handle continuous attributes. De Campos *et al*. [66,67] adapted Ant Colony System for the problem of learning the structure of Bayesian networks, and Pinto *et al*. [68] have recently extended this work. Finally, the work of Socha and Blum [89] for training neural networks by means of ACO is also an example of the application of ACO algorithms to continuous problems.

Bioinformatics Problems

Computer applications to molecular biology (bioinformatics) have originated many $\mathcal{NP}$-hard combinatorial optimization problems. We include in this section general problems that have attracted considerable interest due to their applications to bioinformatics. This is the case of the shortest common supersequence problem (SCSP), which is a well-known $\mathcal{NP}$-hard problem with applications in DNA analysis. Michel and Middendorf [72,73] proposed an ACO algorithm for the SCSP, obtaining state-of-the-art results, in particular, for structured instances that are typically found in real-world applications.

An important problem in bioinformatics is protein folding, that is, the prediction of a protein's structure based on its sequence of amino acids. A simplified model for protein folding is the two-dimensional hydrophobic-polar protein folding problem [114]. Shmygelska and Hoos [74] have successfully applied ACO to this problem and its three-dimensional variant. The performance of the resulting ACO algorithm is comparable to the best existing specialized algorithms for these problems.

Interesting is also the work of Blum *et al*. [78], where they propose a *multilevel framework* based on ACO for the problem of DNA sequencing by hybridization. An earlier proposal of multilevel ACO frameworks is due to Korošec *et al*. [115]. Multilevel techniques [116,117] solve a hierarchy of successively smaller versions of the original problem instance. The solutions obtained at the lowest level of the hierarchy are transformed into solutions for the next higher level, and improved by an optimization algorithm, such as an ACO algorithm.

Other problems in bioinformatics have been successfully tackled by means of ACO algorithms: Korb *et al*. [75,76] considered the flexible protein–ligand docking problem, for which the proposed ACO algorithm reaches state-of-the-art performance, and Benedettini *et al*. [79] recently studied the problem of haplotype inference under pure parsimony. ACO algorithms are sometimes hybridized with Machine Learning techniques. An example is the recent work of Ressom *et al*. [77] on a selection problem in biomarker identification, which combines ACO with support vector machines.

APPLICATIONS TO PROBLEMS WITH NONSTANDARD FEATURES

We review in this section applications of ACO algorithms to problems having additional characteristics such as multiple objective functions, time-varying data, and stochastic information about objective values or constraints. In addition, we mention applications of ACO to network routing and continuous optimization problems.

Multiobjective Optimization

In many real-world problems, candidate solutions are evaluated according to multiple, often conflicting objectives. Sometimes the importance of each objective can be exactly weighted, and hence objectives can be combined into a single scalar value by using, for example, a weighted sum. This is the approach used by Doerner *et al*. [118] for a biobjective transportation problem. In other cases, objectives can be ordered by their relative importance in a lexicographical manner. Gambardella *et al*. [12] proposed a two-colony ACS algorithm for the vehicle routing problem with time windows, where the first colony improves the primary objective and the second colony tries to improve the secondary objective while not worsening the primary one.

When there is no *a priori* knowledge about the relative importance of objectives, the goal usually becomes to approximate the set of Pareto-optimal solutions—a solution is Pareto optimal if no other solution is better or equal for all objectives and strictly better in at least one objective. Iredi *et al*. [80] were among the first to discuss various alternatives for extending ACO to multiobjective problems in terms of Pareto-optimality. They also tested a few of the proposed variants on a biobjective scheduling problem. Another early work is the application of ACO to multiobjective portfolio problems by Doerner *et al*. [81,82]. Later studies have proposed and tested various combinations of alternative ACO algorithms for multiobjective variants of the QAP [83,84], the knapsack problem [85], activity crashing [87], and the biobjective orienteering problem [88]. García-Martínez *et al*. [86] reviewed existing multiobjective ACO algorithms and carried out an experimental evaluation of several ACO variants using the bicriteria TSP as a case study. Angus and Woodward [119] give another detailed overview of available multiobjective ACO algorithms.

Stochastic Optimization Problems

In stochastic optimization problems, data are not known exactly before generating a solution. Rather, because of uncertainty, noise, approximation, or other factors, what is available is stochastic information on the objective function value(s), on the decision variable values, or on the constraint boundaries.

The first application of ACO algorithms to stochastic problems was to the probabilistic TSP (PTSP). In the PTSP, each city has associated a probability of requiring a visit, and the goal is to find an *a priori* tour of minimal expected length over all cities. Bianchi *et al.* [91] and Bianchi and Gambardella [92] proposed an adaptation of ACS for the PTSP. Very recently, this algorithm was improved by Balaprakash *et al.* [93], resulting in a state-of-the-art algorithm for the PTSP. Other applications of ACO to stochastic problems include vehicle routing problems with uncertain demands [94], and the selection of optimal screening policies for diabetic retinopathy [95]. The latter approach builds on the S-ACO algorithm proposed earlier by Gutjahr [120].

Dynamic Optimization Problems

Dynamic optimization problems are those whose characteristics change while being solved. ACO algorithms have been applied to such versions of classical $\mathcal{NP}$-hard problems. Notable examples are applications to dynamic versions of the TSP, where the distances between cities may change or where cities may appear or disappear [98–101]. More recently, Montemanni *et al.* [102] and Donati *et al.* [103] discuss applications of ACS to dynamic vehicle routing problems, reporting good results on both artificial and real-world instances of the problem. Other notable examples of dynamic problems are routing problems in communication networks, which are discussed in the following section.

Communication Network Problems

Some system properties in telecommunication networks, such as the availability of links or the cost of traversing links, are time-varying. The application of ACO algorithms to routing problems in such networks is among the main success stories in ACO. One of the first applications by Schoonderwoerd *et al.* [121] concerned routing in circuit-switched networks, such as classical telephone networks. The proposed algorithm, called *ABC*, was demonstrated on a simulated version of the British Telecom network.

A very successful application of ACO to dynamic network routing is the AntNet algorithm, proposed by Di Caro and Dorigo [96,122]. AntNet was applied to routing in packet-switched networks, such as the Internet. Experimental studies compared AntNet with many state-of-the-art algorithms on a large set of benchmark problems under a variety of traffic conditions [96]. AntNet proved to be very robust against varying traffic conditions and parameter settings, and it always outperformed competing approaches.

Several other routing algorithms based on ACO have been proposed for a variety of wired network scenarios [123,124]. More recent applications of these strategies deal with the challenging class of mobile ad hoc networks (MANETs). Because of the specific characteristics of MANETs (very high dynamics and link asymmetry), the straightforward application of the ACO algorithms developed for wired networks has proven unsuccessful [125]. Nonetheless, an extension of AntNet that is competitive with state-of-the-art routing algorithms for MANETs has been proposed by Ducatelle *et al.* 97, [126]. For recent, in-depth reviews of applications of ACO to dynamic network routing problems, we refer to Refs 127 and 128.

Continuous Optimization Problems

Continuous optimization problems arise in a large number of engineering applications. Their main difference from combinatorial problems, which were the exclusive application field of ACO in the early research efforts, is that decision variables in such problems have a continuous, real-valued domain. Recently, various proposals have been made on how to handle continuous decision variables within the ACO framework [129–131]. In the continuous ACO algorithm proposed by Socha and Dorigo [90], probability density functions, explicitly represented by Gaussian kernel functions, correspond to the pheromone models. Extensions of this approach also exist for mixed-variable—continuous and discrete—problems [132]. A notable application of ACO algorithms for continuous optimization is the training of feed-forward neural networks [89]. Interestingly, there exist also successful applications of ACO to continuous problems that discretize the real-valued domain of the variables. An example is the PLANTS algorithm for the protein–ligand docking problem [76], which combines a discrete ACO algorithm with a local search that works on the continuous domain of the variables.

Industrial Applications

While most research is done on academic applications, commercial companies have started to use ACO algorithms for real-world applications [11]. The company AntOptima (www.antoptima.com) develops and markets ACO-based solution methods for tackling industrial vehicle routing problems. Features common to real-world applications are time-varying data, multiple objectives, or the availability of stochastic information about events or data. Moreover, engineering problems often do not have a mathematical formulation in the traditional sense. Rather, algorithms have to rely on an external *simulator* to evaluate the quality and feasibility of candidate solutions. Examples of applications of ACO relying on simulation are the design [133] and operation [134] of water distribution networks. Other interesting real-world applications are those of Gravel, Price and Gagné [28], who applied ACO to an industrial scheduling problem in an aluminum casting center, and those of Bautista and Pereira [63,135,136], who successfully applied ACO to solve an assembly line balancing problem for a bike line assembly.

CONCLUSIONS

Nowadays, ACO is a well-established metaheuristic applied to a wide range of optimization problems and with hundreds of successful implementations. Several of these implementations have shown to be, at least at the time of their publication, the state-of-the-art for the respective problems tackled, including problems such as vehicle routing, sequential ordering, quadratic assignment, assembly line balancing, open-shop scheduling, and various others. Applications of ACO to dynamic routing problems in telecommunication networks have been particularly successful, probably because several algorithm characteristics match well with the features of the applications.

By analyzing the many available ACO implementations, one can identify ingredients necessary for the successful application of ACO. Firstly, an effective mechanism for iteratively constructing solutions must be available. Ideally, this construction mechanism exploits problem-specific knowledge by using appropriate heuristic information. Secondly, the best performing ACO algorithms have specialized features that allow to carefully control the balance between the exploration of new solutions and the intensification of the search around the best solutions. Such control mechanisms are offered by advanced ACO algorithms such as ACS or $\mathcal{MM}$AS. In fact, the original AC has been abandoned by now in favor of better performing variants. Thirdly, the usage of local search algorithms for improving the solutions constructed by the ants is very successful in practice. Finally, the integration of other techniques such as constraint programming, tree search techniques, or multilevel frameworks often yields a further improvement in performance or increases the robustness of the algorithms.

Further information on ACO and related topics can be obtained by subscribing to the moderated mailing list aco-list, and by visiting the ACO web page (www.aco-metaheuristic.org).

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